ON THE TORSION OF THIN-WALLED MULTICELL SECTIONS*

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SOMMARIO: *Si espone un procedimento per il calcolo a torsione delle sezioni sottili chiuse a connessione multipla ed a spessori sottili, che conduce alla determinazione delle τ in modo rapido e diretto ed evita la scrittura di sistemi di equazioni lineari di ordine elevato.*
Sono riportate alcune applicazioni.

SUMMARY: *Description of a simple method for analysing thin-walled multicell sections which permits the rapid and direct determination of the shear stresses produced by pure twist and obviates the use of high order systems of linear equations.*
Some numerical applications are reported.

1. Introduction.

The exact analysis of the torsional behaviour of elastic beams of multiply—connected (multicell) thin—walled section leads [1] [4] [5] [6] to systems of linear equations with a great number of unknowns.

Iteration and/or electronic computer techniques for solving this kind of systems are available (see [1], pp. 252-254, and [6]).

For very simple sections, approximate methods of analysis permitting more rapid computation, have been proposed [2] [3].

The general method presented in this paper allows exact solution of the problem through a finite number of elementary operations and without introducing a set of linear equations.

By this method, the given section is gradually transformed into others, equivalent but of an increasingly simple type; the solution of the last (biconnected) equivalent section is the starting point for the determination of the τ in the actual section. The calculations are thus much reduced.

2. Generalities.

Let us consider (Fig. 1) a beam subject to simple (uniform) torsion. We assume that its cross-section is thin (1), closed and multiply—connected and that *De Saint Venant's* loading hypotheses hold. Having chosen the three axes of reference with z-axis parallel to the centroidal line of the beam, the analysis may be carried out under the usual hypothesis that $\tau = \tau_z$ is constant in value along each thickness δ.

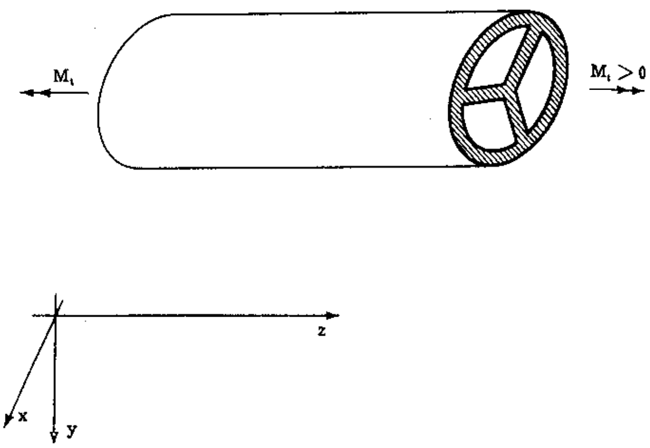

Fig. 1.

Denoting by n_m, n_j and n_b respectively the number of *meshes*, *joints* and *branches* constituting the section (where branch defines a length of the centerline between two consecutive joints and mesh a closed biconnected circuit (2) formed at least by two consecutive branches and not enclosing other branches), it is known [6] that the following relation holds:

$$n_m + n_j = n_b + 1 . \qquad (1)$$

The unknowns of the problem are reduced, on the above hypotheses, to the n_b values of $\tau\delta$ (one in each branch) and to the unit angle of twist ϑ'.

The available equations are:

— that of global equilibrium at rotation about an arbitrary pole P

$$M_t = \Sigma_i{}' \, \tau_i \delta_i l_i h_i , \qquad (2)$$

in which M_t indicates the applied torque, l_i the length

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(1) The section can therefore be thought of as generated by the displacement along a line s (*centerline*) of a segment of length δ (possibly variable along s) normal to s during the motion. The above section is called *thin* if δ is very small compared to the average dimension of a rectangle circumscribed to s and to the radius of curvature of s at every regular point of it: in this case the geometry and the degree of connexion of the section are defined with reference to s.

(2) The above definition of mesh is not the only nor the simplest possible. However, it can be proved that the meshes thus defined form an independent set. In fact, the given section

of the i-th branch and h_i its distance (3) from P. The Σ_i' is extended to all branches of the section;

— the $n_j - 1$ of equilibrium at the joints, of type

$$\Sigma_i'' \ \tau_i \delta_i = 0 \qquad (3)$$

(Σ_i'' extended to the branches which concur in the joint in question);

— finally, the n_m equations of congruence of the meshes, in the form

$$\Sigma_i''' \ \tau_i l_i = 2 \frac{q}{I_p} M_t A_k = 2G\vartheta' A_k \,, \qquad (4)$$

where A_k indicates the area enclosed by mesh k, q and I_p represent the torsion factor and the polar moment of inertia of the section, G is the shear elastic modulus and the Σ_i''' is extended to the branches constituting the mesh (4).

The system of $(n_b + 1)$ Eqs. (2), (3) and (4) solves the problem, and is the one usually adopted.

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can be built-up by putting together, one at each time, the single meshes. If an intermediate configuration of the section consists

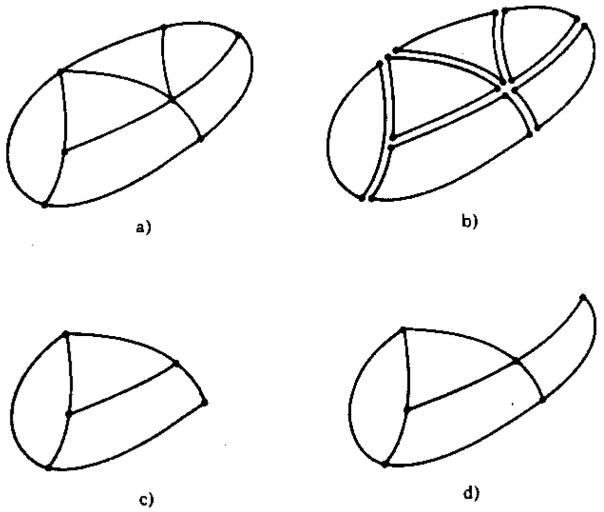

Fig. 2.

of n independent meshes, the next configuration consists of $n + 1$ independent meshes, because the $(n + 1)$-th mesh contains at least one side not in common with any other. Therefore, since the starting triconnected section is made of independent meshes, so is the complete one (Fig. 2).

(3) Eq. (2) holds in the particular case of straight branches. On the more general hypothesis that the various branches are curved, calling δ_i^* a reference thickness on i and τ_i^* the corresponding τ, Eq. (2) is written [6]:

$$M_t = \Sigma_i' \ \tau_i^* \delta_i^* l_i^* h_i^* \,, \qquad (2')$$

where:

$$l_i^* = \delta_i^* \int_i \frac{dl}{\delta_i} \,, \qquad h_i^* = \frac{1}{l_i^*} \int_i h_i dl \,.$$

For the signs of the various terms in Eqs. (2) and (2′), see [5] and [6].

(4) Eqs. (3) and (4), in the case of curvilinear branches, are written respectively [6]:

$$\Sigma_i'' \ \tau_i^* \delta_i^* = 0 \qquad (3')$$

and

$$\Sigma_i''' \ \tau_i^* l_i^* = 2G\vartheta' A_k \,. \qquad (4')$$

A lower degree system can be obtained by substitution of Eqs. (3) into Eqs. (4), or selecting auxiliary unknowns [1]: the number of equations is then reduced to $(n_m + 1)$. Let us introduce, for each branch i, the *shear flow* $t_i = \tau_i \delta_i$, constant along the branch itself, and the non-dimensional quantity

$$\lambda_i = \frac{l_i}{\delta_i} \,,$$

which may be called *slenderness*; Eqs. (2), (3) and (4) assume then the form:

$$\Sigma_i' \ t_i l_i h_i = M_t \qquad (2'')$$

$$\Sigma_i'' \ t_i = 0 \qquad (3'')$$

$$\Sigma_i''' \ t_i \lambda_i = 2G\vartheta' A_k \qquad (4'')$$

and all together constitute a system which will be called *complete*, giving the name of *reduced system* to that formed by Eqs. (3″) and (4″).

It is possible to obtain in two steps the solving t_i values of the problem, and therefore the τ_i. Firstly a torque

$$\overline{M}_t = \frac{I_p}{2q}$$

is applied to the section.

If $\bar{t}_i$ are the shear flows corresponding to $\overline{M}_t$, Eqs. (4″) become:

$$\Sigma_i''' \ \bar{t}_i \lambda_i = A_k \qquad (4''')$$

and, together with

$$\Sigma_i'' \ \bar{t}_i = 0 \qquad (3''')$$

are sufficient to determine the $\bar{t}_i$; Eq. (2″) then furnishes:

$$\overline{M}_t = \Sigma_i' \ \bar{t}_i l_i h_i \,. \qquad (2''')$$

Finally, by multiplying the $\bar{t}_i$ values by the ratio $M_t/\overline{M}_t$, we get to the t_i values for the torque applied (5). The problem is thus reduced to determining the $\bar{t}_i$; and for this purpose the reduced system (3‴) (4‴) is necessary and sufficient. To solve it indirectly, we may use some transformations of the assigned section, such that the quantities $\bar{t}$ in the branches not affected by the transformation remain constant and equal to the solutions of the reduced system (6).

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(5) From (2″) one obtains:

$$2G\vartheta' = \frac{M_t}{\Sigma_i' \ \bar{t}_i l_i h_i} \,,$$

from which

$$t_i = \frac{\bar{t}_i M_t}{\Sigma_i' \ \bar{t}_i l_i h_i} \,.$$

(6) The procedure is the same even in the case of section having curvilinear branches, in which case one puts:

$$t_i = \tau_i^* \delta_i^* \,, \qquad \lambda_i = \int_i \frac{dl}{\delta_i} \,.$$

Finally, it may be interesting to note the analogy between the current distribution problem for an electrical resistive network and the one analyzed above.

For a network with the same topological configuration of the assigned multicell section, the well—known Kirchhoff equations may be written as follows:

joint Eq.

$$\Sigma i_j = 0 \tag{a}$$

mesh Eq.

$$\Sigma i_j r_j = e_k , \tag{b}$$

where r_j and i_j are respectively the resistance of and the current in the j-th branch, and e_k is the electromotive resultant force in the k-th mesh.

The analogy can be easily established by the correspondence rule:

$$\bar{t} \rightarrow i$$
$$\lambda \rightarrow r$$
$$A \rightarrow e ,$$

and may be used for the analogical model of the section, as in many others torsion problems [7].

3. Branches in series.

Let us examine the detail in Fig. 3a. Branches a and b, having in common joint k in which no other branches

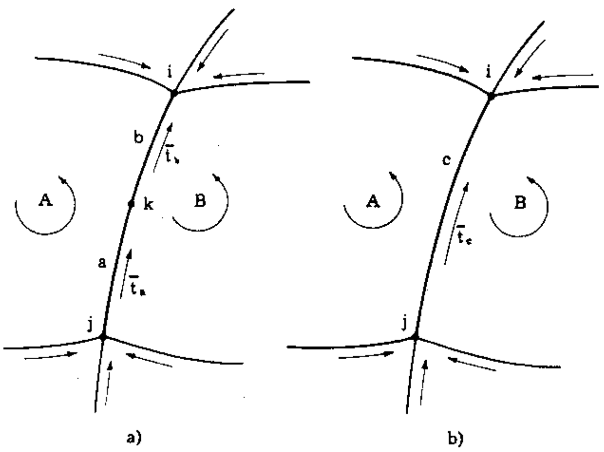

Fig. 3.

concur, are said to be *in series*. To replace them by a single branch c, of slenderness λ_c such that the flow distribution in the other branches of the section remains unchanged, it is necessary and sufficient that:

$$\lambda_c = \lambda_a + \lambda_b \tag{5}$$

which in such a case results in:

$$\bar{t}_a = \bar{t}_b = \bar{t}_c .$$

To demonstrate this result, it is sufficient to write the equations (7) of joints i, j, k, and of meshes A and B before the transformation (these equations are the only ones of the reduced system in which branches a and b appear) and compare them with the corresponding ones for the transformed section (Fig. 3b).

It is also easy to demonstrate the inverse proposition, according to which any branch c may be interrupted by a joint k without varying the flow distribution in the remaining section, provided that the slendernesses of the new and the old branch are related by Eq. (5), and the extension of same Eq. (5) to the case of several branches in series.

4. Branches in parallel.

Two branches ending in the same joints are said to be *in parallel*; furthermore, it is supposed for the time being that they form a mesh. To two branches of such kind (Fig. 4a) it is be possible to substitute only one connecting the same joints: the section is thus modified even in the

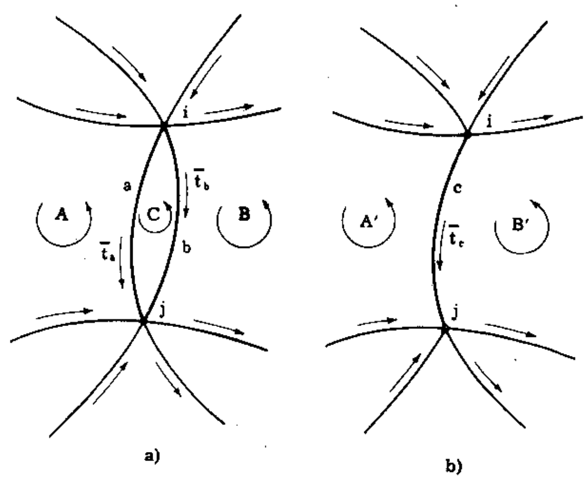

Fig. 4.

areas of meshes adjacent to the suppressed one. The conditions of equivalence, with the symbols and directions of Fig. 4, are written:

$$\frac{1}{\lambda_c} = \frac{1}{\lambda_a} + \frac{1}{\lambda_b} ,$$

$$A_{A'} = A_A + A_C \frac{\lambda_c}{\lambda_b} , \tag{6.a}$$

$$A_{B'} = A_B + A_C \frac{\lambda_c}{\lambda_a} ,$$

whence

$$\bar{t}_c = \bar{t}_a + \bar{t}_b . \tag{6.b}$$

(7) In Fig. 3 and in the two following ones the positive directions of the circuitation and of the t_i are indicated. In Eqs. (3′′) the outward flow from the joint [6] is assumed positive.

It is sufficient, to arrive at Eqs. (6), to compare the equations at joints i and j and at meshes A and B of the section of Fig. 4a with those relative to section of Fig. 4b, equalling the varied terms, and to keep in mind the equation at mesh C

$$\bar{t}_a \lambda_a - \bar{t}_b \lambda_b = A_C ,$$

and the geometrical condition

$$A_A + A_B + A_C = A_{A'} + A_{B'} .$$

Similarly, it can be demonstrated that a branch c of slenderness λ_c ending in the joints i and j may be substituted in equivalence for the rest of the section by two branches, inserted between the same joints, which enclose an area A_C and to one of which is assigned the slenderness; then Eqs. (6-a) and the following holds:

$$\bar{t}_a = \bar{t}_c \frac{\lambda_c}{\lambda_a} + A_C \frac{\lambda_c}{\lambda_a \lambda_b} , \qquad \bar{t}_b = \bar{t}_c \frac{\lambda_c}{\lambda_b} - A_C \frac{\lambda_c}{\lambda_a \lambda_b} . \tag{6.c}$$

The supplied relations might be generalized to the case of more branches connected by the same joints, but they would become highly complicated due to the presence of a greater number of areas to take into account; it is therefore convenient to operate on a couple of branches at a time through Eqs. (6).

5. Delta-star transformation.

The whole of three branches having a common end h in which other branches do not concur is called *star* of center h; a three branch mesh is given the name of *triangle* or *delta*: it is possible to substitute to a given star a delta enclosing an assigned area and viceversa.

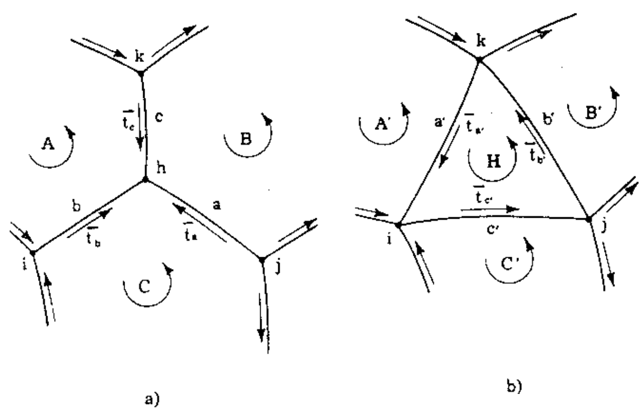

Fig. 5.

With the symbols and the directions of Fig. 5, the equivalence conditions, for the first transformation (star → delta), are written as follows:

$$\bar{t}_a + \bar{t}_b + \bar{t}_c = 0$$

$$\lambda_{a'} \bar{t}_{a'} + \lambda_{b'} \bar{t}_{b'} + \lambda_{c'} \bar{t}_{c'} = A_H$$

$$\bar{t}_a = \bar{t}_{b'} - \bar{t}_{c'}$$

$$\bar{t}_b = \bar{t}_{c'} - \bar{t}_{a'}$$

$$\bar{t}_c = \bar{t}_{a'} - \bar{t}_{b'}$$

$$A_A + A_B + A_C = A_{A'} + A_{B'} + A_{C'} + A_H$$

$$\lambda_b \bar{t}_b - \lambda_c \bar{t}_c - A_A = - \lambda_{a'} \bar{t}_{a'} - A_{A'}$$

$$\lambda_a \bar{t}_a - \lambda_b \bar{t}_b - A_C = - \lambda_{c'} \bar{t}_{c'} - A_{C'} ,$$

whence the characteristics of the equivalent triangle are obtained:

$$\lambda_{a'} = \frac{\lambda_b \lambda_c}{\lambda_0} \qquad A_{A'} = A_A - \frac{\lambda_0}{\lambda_a} A_H$$
$$\lambda_{b'} = \frac{\lambda_c \lambda_a}{\lambda_0} \qquad A_{B'} = A_B - \frac{\lambda_0}{\lambda_b} A_H \tag{7.a}$$
$$\lambda_{c'} = \frac{\lambda_a \lambda_b}{\lambda_0} \qquad A_{C'} = A_C - \frac{\lambda_0}{\lambda_c} A_H$$

where

$$\frac{1}{\lambda_0} = \frac{1}{\lambda_a} + \frac{1}{\lambda_b} + \frac{1}{\lambda_c} .$$

The values of the flows are finally given by:

$$\bar{t}_{a'} = - \frac{\lambda_0}{\lambda_c} \bar{t}_b + \frac{\lambda_0}{\lambda_b} \bar{t}_c + \frac{\lambda_0^2}{\lambda_a \lambda_b \lambda_c} A_H$$
$$\bar{t}_{b'} = - \frac{\lambda_0}{\lambda_a} \bar{t}_c + \frac{\lambda_0}{\lambda_c} \bar{t}_a + \frac{\lambda_0^2}{\lambda_a \lambda_b \lambda_c} A_H \tag{7.b}$$
$$\bar{t}_{c'} = - \frac{\lambda_0}{\lambda_b} \bar{t}_a + \frac{\lambda_0}{\lambda_a} \bar{t}_b + \frac{\lambda_0^2}{\lambda_a \lambda_b \lambda_c} A_H .$$

Similarly, a given delta can be transformed into an equivalent star provided that:

$$\lambda_a = \frac{\lambda_{b'} \lambda_{c'}}{\lambda_1} \qquad A_A = A_{A'} + \frac{\lambda_{a'}}{\lambda_1} A_H$$
$$\lambda_b = \frac{\lambda_{c'} \lambda_{a'}}{\lambda_1} \qquad A_B = A_{B'} + \frac{\lambda_{b'}}{\lambda_1} A_H \tag{8.a}$$
$$\lambda_c = \frac{\lambda_{a'} \lambda_{b'}}{\lambda_1} \qquad A_C = A_{C'} + \frac{\lambda_{c'}}{\lambda_1} A_H$$

with

$$\lambda_1 = \lambda_{a'} + \lambda_{b'} + \lambda_{c'} ,$$

and with:

$$\bar{t}_a = \bar{t}_b' - \bar{t}_c' ; \quad \bar{t}_b = \bar{t}_c' - \bar{t}_a' ; \quad \bar{t}_c = \bar{t}_a' - \bar{t}_b' . \tag{8.b}$$

We explicitely point out that the star → delta and delta → star transformations just introduced cannot be immediately applied to meshes or stars with more than three branches, apart from particular cases [8].

6. Applications.

Through the repeated application of Eqs. (5), (6), (7) and (8) any section can be reduced to the simplest, biconnected; the flow calculated on the latter may afterwards subdivide itself, through the inverse sequence of transformations, into the original branches, thus solving the reduced system.

A first example is developed in Fig. 6, which schematizes, for the purposes of calculation, a wing section whose geometrical characteristics are summarized in Tables 1 and 2. In the tables and in the following pages, the joints, branches and meshes are labelled as indicated on the same Fig. 6. Pole P, from which distances h are calculated, coincides with joint 1. The applied twisting torque is equal to

$$M_t = 100{,}000 \text{ kgm.}$$

Firstly, branches 1, 9 and 18 are substituted by only one of slenderness

$$\lambda_1 + \lambda_9 + \lambda_{18} = 326.667 ;$$

the areas remain unaltered. Similarly, branches 8, 17

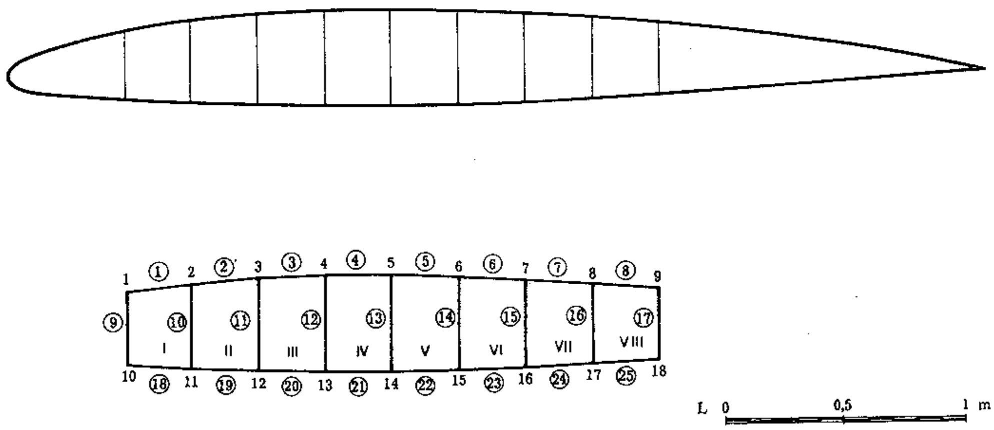

Fig. 6.

TABLE 1

Branch	l (m)	δ (m)	λ	h (m)
1	0.265	0.0030	88.333	0.000
2	0.270	0.0030	90.000	0.020
3	0.270	0.0030	90.000	0.050
4	0.270	0.0030	90.000	0.070
5	0.270	0.0030	90.000	0.100
6	0.270	0.0030	90.000	0.135
7	0.270	0.0030	90.000	0.160
8	0.270	0.0030	90.000	0.190
9	0.300	0.0020	150.000	0.000
10	0.350	0.0005	700.000	0.265
11	0.380	0.0005	760.000	0.535
12	0.400	0.0005	800.000	0.805
13	0.395	0.0005	790.000	1.075
14	0.380	0.0005	760.000	1.345
15	0.355	0.0005	710.000	1.615
16	0.320	0.0005	640.000	1.885
17	0.285	0.0020	142.500	2.155
18	0.265	0.0030	88.333	0.305
19	0.270	0.0030	90.000	0.315
20	0.270	0.0030	90.000	0.325
21	0.270	0.0030	90.000	0.345
22	0.270	0.0030	90.000	0.365
23	0.270	0.0030	90.000	0.380
24	0.270	0.0030	90.000	0.395
25	0.270	0.0030	90.000	0.395

TABLE 2

Mesh	Area (m²)	Mesh	Area (m²)
I	0.0888	V	0.1070
II	0.1010	VI	0.1000
III	0.1080	VII	0.0920
IV	0.1080	VIII	0.0810

and 25 are substituted by the equivalent one, of slenderness

$$\lambda_8 + \lambda_{17} + \lambda_{25} = 322.500$$

(Fig. 7a). Subsequently, the branches in parallel between joints 2 and 10 are substituted by a single branch, of slenderness

$$\frac{1}{\dfrac{1}{326.667} + \dfrac{1}{700}} = 222.727 ,$$

and the area A_{II} becomes:

$$A_{II'} = A_{II} + \frac{222.727}{326.667} A_I = 0.1615 \text{ m}^2.$$

This branch is in series with branches 2 and 19, and therefore we can put:

$$\lambda_{eq.} = 222.727 + \lambda_2 + \lambda_{19} = 402.727.$$

It is possible to operate in the same way with respect to the end of the section on the right-hand side (Fig. 7b), in order to eliminate mesh VIII.

Analogous steps lead to the sections in Figs. 7c and 7d, and from these, through the elimination of the right-hand mesh, to that in Fig. 7e. If $\overline{t}_{21} = \overline{t}_4$ is the flow in the latter, we have:

$$\overline{t}_{21} = \frac{0.3915}{754.362} = 518.984 \cdot 10^{-6} \text{ m}^2.$$

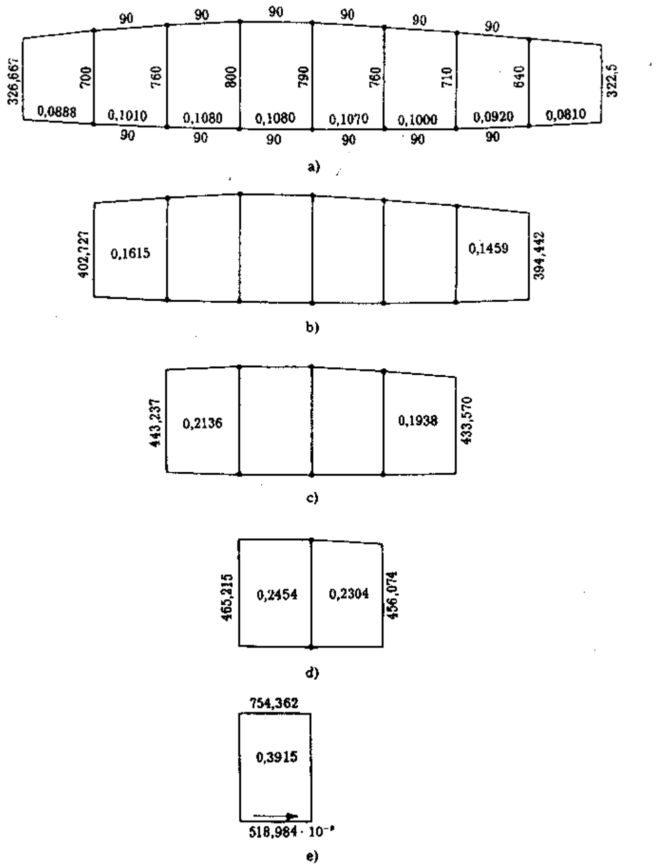

Fig. 7.

Getting back to the previous section (Figs. 7d and 8a), it is possible to divide $\overline{t}_{21}$ into $\overline{t}_{13}$ and $\overline{t}_{22}$, according to (6), whose coefficients have already been calculated in the direct transformation. We thus obtain:

$$\overline{t}_{13} = 5.086 \cdot 10^{-6} \text{ m}^2$$

$$\overline{t}_{22} = 513.916 \cdot 10^{-6} \text{ m}^2.$$

Similarly, it is possible to go to section of Fig. 8c and the final one of Fig. 8d, by which time the solution of the reduced system is complete.

The equilibrium Eq. (2''') yields:

$$\overline{M}_t = 739.812 \cdot 10^{-6} \text{ kg/m}^3,$$

and the actual stresses under the applied torque are given by:

$$\tau_i = \frac{\overline{t}_i}{\delta_i} \frac{M_t}{\overline{M}_t}.$$

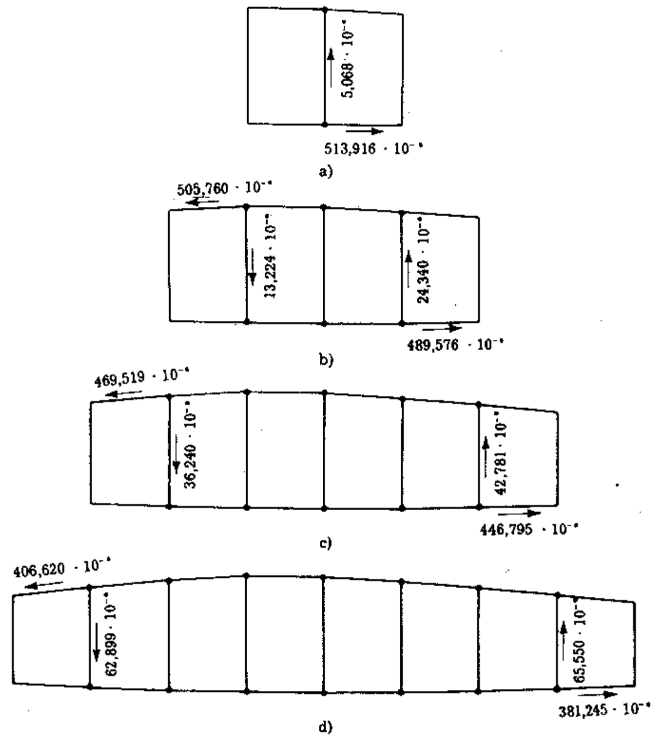

Fig. 8.

The values in the various branches are shown in Table 3 and plotted in Fig. 9.

A second example of application is offered by the calculation of the box section of Fig. 10, whose characteristics (distances h are relative to joint 1) are shown in Tables 4 and 5. The section is subjected to a torque

$$M_t = 50{,}000 \text{ kgm}.$$

The successive transformations for reducing the section to the biconnected one are represented in Fig. 11, in which the numerical values obtained at every step are

TABLE 3

Branch	τ (kg/cm²)	Branch	τ (kg/cm²)
1	1832	14	658
2	2115	15	1157
3	2279	16	1772
4	2338	17	2577
5	2316	18	1832
6	2206	19	2115
7	2013	20	2279
8	1718	21	2338
9	2748	22	2316
10	1700	23	2206
11	980	24	2013
12	358	25	1718
13	137		

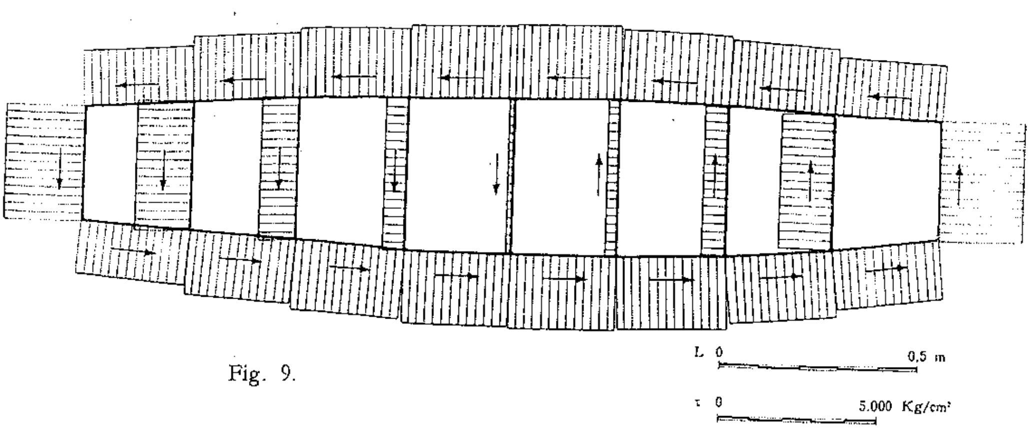

Fig. 9.

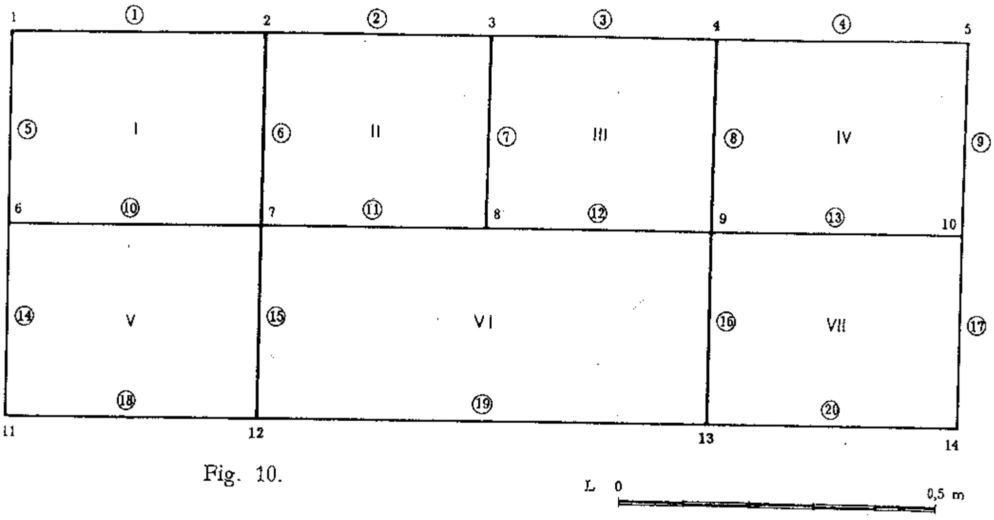

Fig. 10.

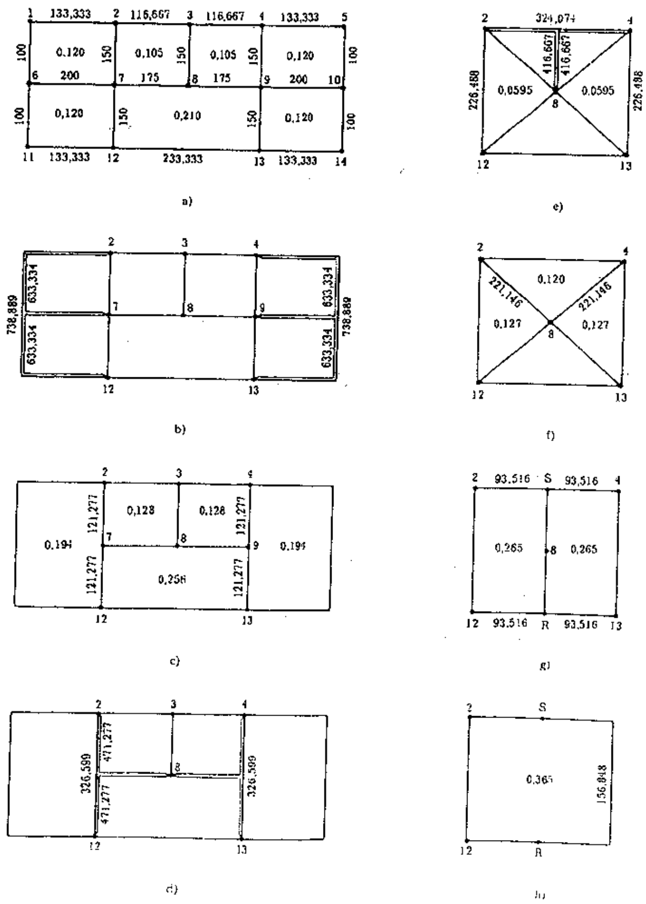

Fig. 11.

TABLE 4

Branch	l (m)	δ (m)	λ	b (m)
1	0.40	0.003	133.333	0.00
2	0.35	0.003	116.667	0.00
3	0.35	0.003	116.667	0.00
4	0.40	0.003	133.333	0.00
5	0.30	0.003	100.000	0 00
6	0.30	0.002	150.000	0.40
7	0.30	0.002	150.000	0.75
8	0.30	0.002	150.000	1.10
9	0.30	0.003	100.000	1.50
10	0.40	0.002	200.000	0.30
11	0.35	0.002	175.000	0.30
12	0.35	0.002	175.000	0.30
13	0.40	0.002	200.000	0.30
14	0.30	0.003	100.000	0.00
15	0.30	0.002	150.000	0.40
16	0.30	0.002	150.000	1.10
17	0.30	0.003	100.000	1.50
18	0.40	0.003	133.333	0.60
19	0.70	0.003	233.333	0.60
20	0.40	0.003	133.333	0.60

ABLE 5

Mesh	Area (m²)	Mesh	Area (m²)
I	0.120	V	0.120
II	0.105	VI	0.210
III	0.105	VII	0.120
IV	0.120		

also shown. To sum up, the steps are as follows:

— substitution of branches 1 and 5 and of branches 14 and 18 by the equivalent series (similarly for those symmetrical with respect to the axis 3÷8);

— substitution of the star with center 6 and vertices 2, 7 and 12 by a delta of nil area (and analogous transformation as to the symmetrical part) (see Fig. 11b);

— composition in parallel of the branches between joints 2—7 and 7—12 (and symmetrical) (as in Fig. 11c);

— substitution of the star with center 7 and vertices 2, 8 and 12 (and symmetrical) by a triangle of nil area (Fig. 11d);

— composition in parallel of the branches between joints 2—12 and 4—13;

— substitution of the star with center 3 by a nil area triangle (Fig. 11e);

— composition in parallel of the branches between joints 2—8 and 4—8 (Fig. 11f);

— transformation of the triangles with vertices 2—4—8 and 12—13—8 into stars of centers, respectively, S and R (Fig. 11g);

— composition in series of the branches S — 8 and 8 — R and of the branches R — 13, 13 — 4 and 4 — S;

— composition in parallel of the branches derived between joints S and R (Fig. 11h).

For the last section, it is easy to obtain:

$$\bar{t} = 639.983 \cdot 10^{-6}\ \mathrm{m}^2$$

and from this value, through the above-mentioned transformations effected in inverse order, it is easy to reach, as shown in Fig. 12, the stress distribution in the original section.

From Eq. (2‴) we then have:

$$\bar{M}_t = 1188.750 \cdot 10^{-6}\ \mathrm{kg/m^3}$$

and there follows, as in the preceding case, the flows and the actual stresses, the latter being shown in Table 6 and plotted in the Fig. 13.

It is worth noting that the calculation, worked out in this way for the purpose of exemplification, could have been done much more quickly once the nil values of the stresses have been recognized on the basis of simple considerations of symmetry.

TABLE 6

Branch	τ (kg/cm²)	Branch	τ (kg/cm²)
1	840	11	0
2	1024	12	0
3	1024	13	0
4	840	14	840
5	840	15	277
6	277	16	277
7	0	17	840
8	277	18	840
9	840	19	1024
10	0	20	840

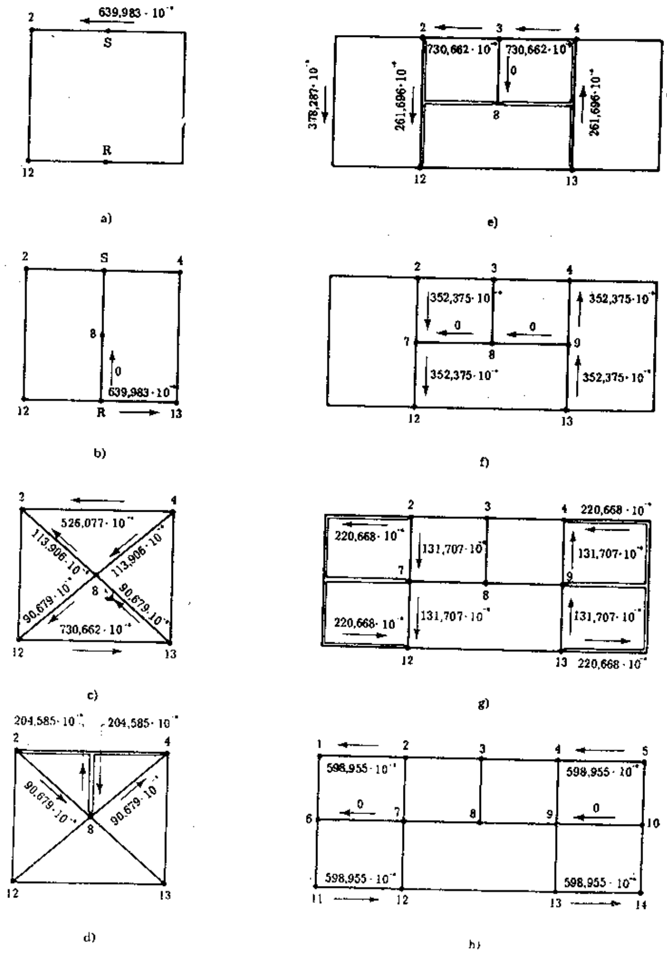

Fig. 12.

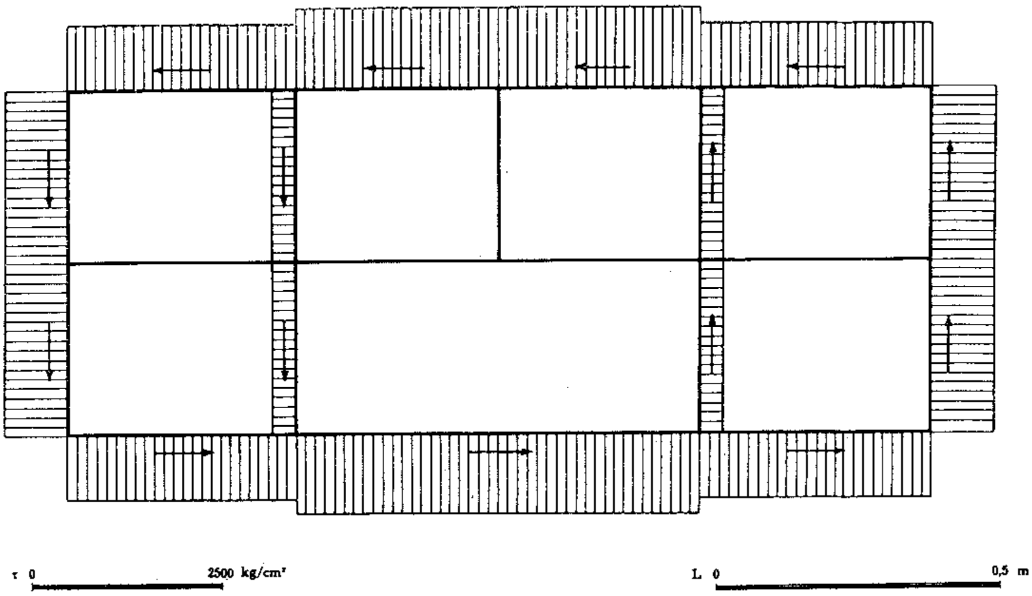

Fig. 13.

7. Conclusions.

We consider that the method described, thanks to the reduced number of calculations and to the simplicity of its procedure by schemes, is well suited to the analysis of quite complex structures and is undoubtedly preferable to methods involving systems of linear equations, which are of a very high order even in the simplest cases.

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REFERENCES

[1] S. P. Timoshenko, *Strenght of Materials*, Vol. II, Van Nostrand, N. Y., 1956.

[2] E. H. Mansfield, *Torsional Stresses in Multiwebbed Rectangular Cylinders*, Aircraft Engineering, Jan. 1953.

[3] F. W. Niedenfuhr, *Torsion Analysis of Multicell Tubes*, J. Aerospace Sci., Apr. 1960.

[4] V. Franciosi, *Scienza delle Costruzioni*, Vol. II, Liguori, Napoli, 1962.

[5] V. Franciosi, *Le travi a sezione sottile*, Rend. del Corso di Perfezionamento per le costr. in c. a., Vol. XVII, Tamburini, Milano, 1965.

[6] R. Sparacio and F. Maceri, *Sul calcolo a torsione delle sezioni sottili chiuse pluriconnesse*; Rend. Accad. di Sc. Fis. e Mat., Ser. 4, Vol. XXXII, Napoli, 1965.

[7] M. Hetenyi, *Handbook of Experimental Stress Analysis*, Wiley, N. Y., 1950.

[8] E. A. Guillemin, *Introductory Circuit Theory*, Wiley, N. Y., 1953.