

Testing Galileo's
“Universality of Free Fall” (UFF)
to higher and higher accuracy.
Why and how?

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Università di Roma “Tor Vergata”, 7 Dicembre 2007

Galileo: Aristoteles cannot be right!

Aristoteles view: the free fall velocity of bodies is proportional to their weight

Galileo's simple logical argument...

“ If then we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will be partly retarded by the slower and the slower will be somewhat hastened by the swifter... hence the heavier body (made by the two tied together) moves with less speed than the lighter;
an effect which is contrary to your supposition.”

Ex: two bricks tied together to form a single brick do obviously fall at the same speed as each brick separately, while according to Aristoteles should fall faster because they are twice as heavy...

How did Galileo come to formulate the UFF?

- Aristoteles view cannot be right!
- Experimental proof is needed!
- Guess: the difference in time of fall may be due to air resistance (...test guess by amplifying resistance of medium)
- Difficulties with experiments based on dropping masses from a height ...too fast... too much air resistance...
- Inclined plane experiments
- Pendulum experiments

Experimental proof is needed!

“ The facts set forth by me up to this point and, in particular, the one which shows that difference of weight, even when very great, is without effect in changing the speed of falling bodies, so that as far as weight is concerned they all fall with equal speed: **this idea is, I say, so new, and at first glance so remote from fact that if we do not have the means of making it just as clear as sunlight, it had better not be mentioned;** but having once allowed it to pass my lips I must neglect no experiment or argument to establish it”

This is what makes the difference w.r.t Philopponus (6th century) and Benedetti (1553) who had questioned Aristoteles view before Galileo...

Galileo's handwritten statement of the Universality of free fall

una palla d'oro nel fine della scesa di 400. braccia. Non frequerà una di rame di 420. braccia.
Veduto, dico, questo cascai in opinione che se si levasse totalm^{te} la resistenza del mezzo tutte
le materie

24
le materie descenderebbero con eguali velocità.
Simpl. Gran detto è questo sig. Salvi. Io non crederò mai che nell'istesso vacuo; se pur vi è.
Luna si muovesse così veloce come un pezzo di piombo.

Veduto, dico, questo cascai in opinione che se si levasse
totalmente la resistenza del mezzo tutte le materie
descenderebbero con uguali velocità.

Having observed this I came to the conclusion that in a medium totally devoid of
resistance all bodies **[substances]** would fall with the same speed.

DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue scienze

Attenenti alla
MECANICA & i MOVIMENTI LOCALI;
del Signor

GALILEO GALILEI LINCEO,
Filosofo e Matematico primario del Serenissimo
Grand Duca di Toscana.

Con una Appendice del centro di gravità d'alcuni Solidi.



IN LEIDA,
Appresso gli Elsevirii. M. D. C. XXXVIII.



Galileo's birthplace in Pisa

Dialogues Concerning
**TWO NEW
SCIENCES**
GALILEO GALILEI



TRANSLATED BY
Henry Crew & Alfonso de Salvio

WITH AN INTRODUCTION BY
Antonio Favaro

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Galileo's pendulum test of the UFF

Accordingly I took two balls, one of lead and one of cork, the former more than a hundred times heavier than the latter, and suspended them by means of two equal fine threads, each four or five cubits long. Pulling each ball aside from the perpendicular, I let them go at the same instant, and they, falling along the circumferences of circles having these equal strings for semi-diameters, passed beyond the perpendicular and returned along the same path. This free vibration [per lor medesima le andate e le tornate] repeated a hundred times showed clearly that the heavy body maintains so

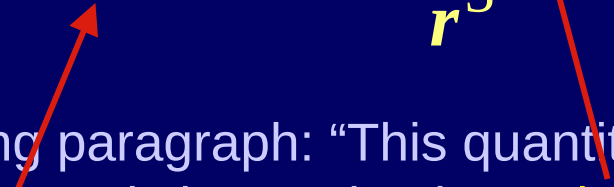
[129]

nearly the period of the light body that neither in a hundred swings nor even in a thousand will the former anticipate the latter by as much as a single moment [minimo momento], so perfectly do they keep step. We can also observe the effect of the medium which it offers to motion

Most probably the first null experiment in Physics!

without altering the frequency of either; even when the arc traversed by the cork did not exceed five or six degrees while that of the lead was fifty or sixty, the swings were performed in equal times.

In the gravitational field of a source body (e.g. the Earth):

$$m_i \ddot{\vec{g}} = - \frac{GM_{\oplus} m_g}{r^3} \vec{r}$$


PRINCIPIA opening paragraph: “This quantity that I mean hereafter under the name of ... **mass** ... is known by the **weight** ... for it is proportional to the weight as I have found by experiments on pendulums, very accurately made... ” (Equivalence of inertial-to-gravitational mass: Equivalence Principle)

$$m_g = m_i \Rightarrow \eta \equiv \frac{\Delta g}{g} = 0$$

Universality of Free Fall (UFF)

Einstein's view (I)

Einstein (1907): “hypothesis of complete physical equivalence” between a gravitational field and an accelerated reference frame: In a freely falling system all masses fall equally fast, hence gravitational acceleration has no local dynamical effects (“Weak Equivalence Principle” - WEP)

An observer inside Einstein elevator will not be able to tell, before hitting the ground, whether he is moving with an acceleration g in empty space, far away from all masses, or else he is falling in the vicinity (height of fall $\ll$ Earth radius) of a body (the Earth) whose local gravitational acceleration is also g (and in the same direction).

The experimental consequence is the Universality of Free Fall (UFF):

$$\eta \equiv \frac{\Delta g}{g} = 0$$

... unique to gravity!

- “Einstein Equivalence Principle (EEP)” (assumes WEP): all gravitational effects are (globally) replaced by the metric of a curved, 4-dimension space-time...

General Relativity is based on WEP

WEP $\Rightarrow$ UFF: should bodies of different composition fall with different accelerations, the elevator and the test mass inside it would generally fall with different accelerations and the observer would be able to tell that he is close to the surface of the Earth and not in an accelerated frame in empty space.

UFF experiments test the foundations of GR

a deviation from UFF would require a modification of GR

L'inizio del secolo scorso ha visto un succedersi di idee e di scoperte che hanno influenzato lo sviluppo della fisica fino ai nostri giorni:

- * Nel **1900** Planck introdusse la costante **h** , che porta il suo nome, dando inizio alla rivoluzione quantistica (con un contributo importante di Einstein, nel 1905 (EF));
- * Nel **1905** Einstein partì dall'invarianza della velocità della luce nel vuoto, **c** , per arrivare alla teoria della relatività ristretta e alla famosa equazione: $E = mc^2$;
- * Nel **1915** Einstein partì dall'universalità della caduta libera, per arrivare a una teoria geometrica della gravitazione, la relatività generale, in cui la costante di Newton, **G** , determina la misura in cui la materia curva lo spazio-tempo

Nella prima parte della sua vita scientifica Einstein aveva dunque giocato magistralmente con queste costanti:

$c \Rightarrow$ Relatività Ristretta;

$(c + h) \Rightarrow$ Effetto Fotoelettrico;

$(c + G) \Rightarrow$ Relatività Generale.

Successivamente si pose il problema di come combinare tutte e tre le costanti unificando così

Elettromagnetismo e Gravitazione

Ma lui stesso aveva «gettato la prima pietra» proponendo:

- una versione quantistica del primo (EF: $E_\gamma = h f$);
- una puramente classica della seconda (RG).

Come risolvere un tale contrasto?

Nè Einstein nè altri ci riuscirono

« Devo sembrare uno struzzo che seppellisce la testa nella
sabbia relativistica per non far fronte ai **quanti diabolici** »
(Einstein, 1954)

Einstein preferì la sabbia relativistica ai quanti diabolici:

"Dio non gioca ai dadi!"

alludendo al carattere non deterministico della meccanica
quantistica

Il tentativo di Einstein era quello di unificare
la nostra comprensione del mondo

macroscopico e microscopico,
classico e quantistico
«infinitamente» grande
e
«infinitamente» piccolo

Ma cosa intende oggi il fisico per
«infinitamente» ?

Già nel 1899 Planck aveva osservato che le 3 costanti G, c, h , possono essere combinate per darne altre tre, più intuitive, chiamate da allora:

1. la lunghezza di Planck L_P ,
2. il tempo di Planck T_P ,
3. la massa di Planck M_P .

In formule e numeri:

$$L_P = \sqrt{\frac{G h}{c^3}} \sim 10^{-35} m$$

$$T_P = \frac{L_P}{c} \sim 10^{-43} s$$

$$M_P = \sqrt{\frac{hc}{G}} \sim 10^{-8} Kg$$

Le scale «minime» di lunghezza e di tempo oggi concepibili in fisica sono appunto:

$$L_P = \sqrt{\frac{G h}{c^3}} \sim 10^{-35} m$$

$$T_P = \frac{L_P}{c} \sim 10^{-43} s$$

Quali sono invece le «massime»? Legge di Hubble:

$$\Delta\lambda = \frac{d\lambda}{L_H}$$

$$L_H \sim 10^{26} m, T_H \sim 10^{18} s$$

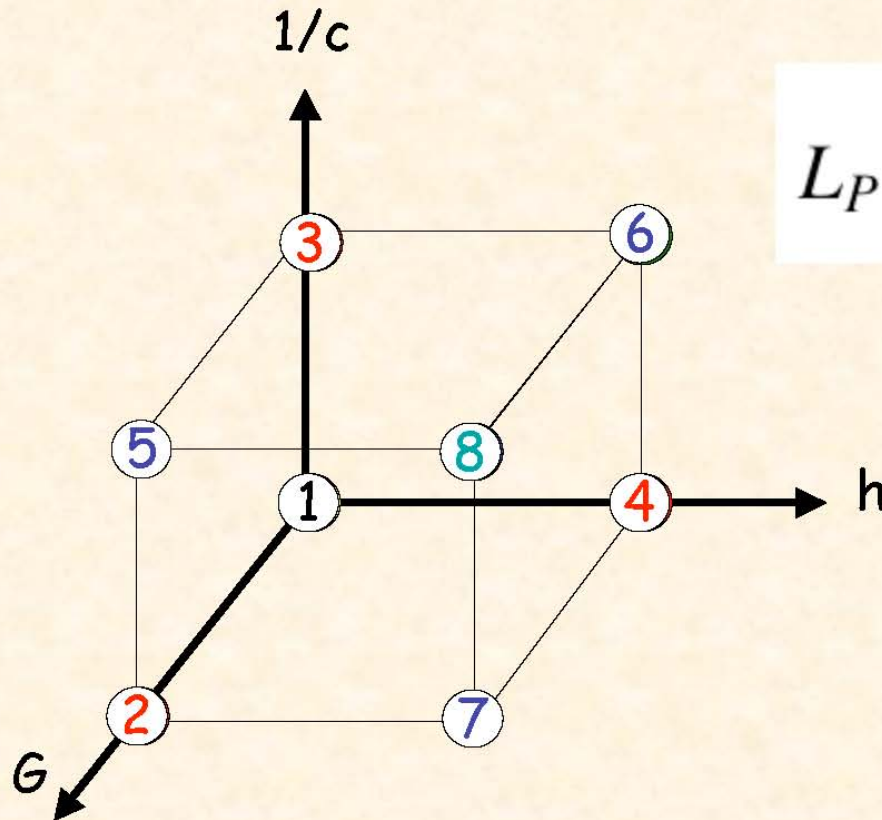
~ dimensioni, età dell'universo osservabile

Il rapporto:

$$\frac{L_H}{L_P} = \frac{T_H}{T_P} \sim 10^{61}$$

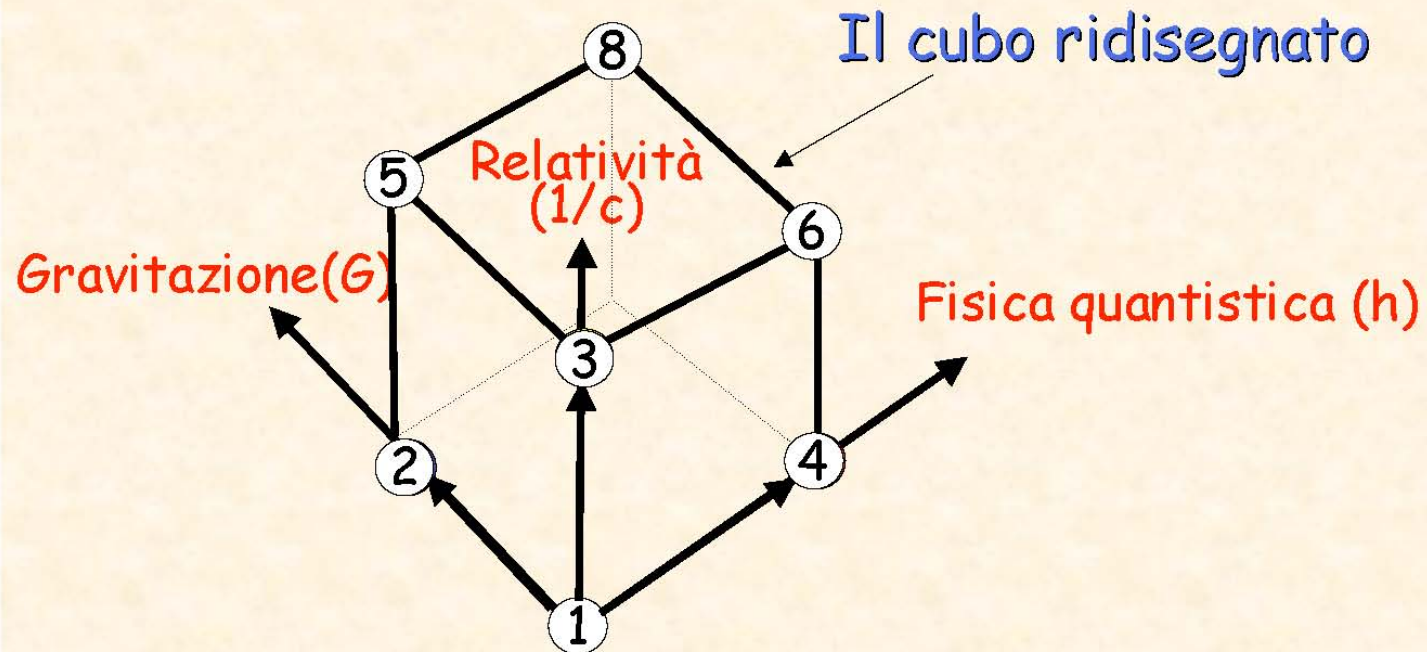
è una buona approssimazione dell'infinito..

Un cubo « metateorico »

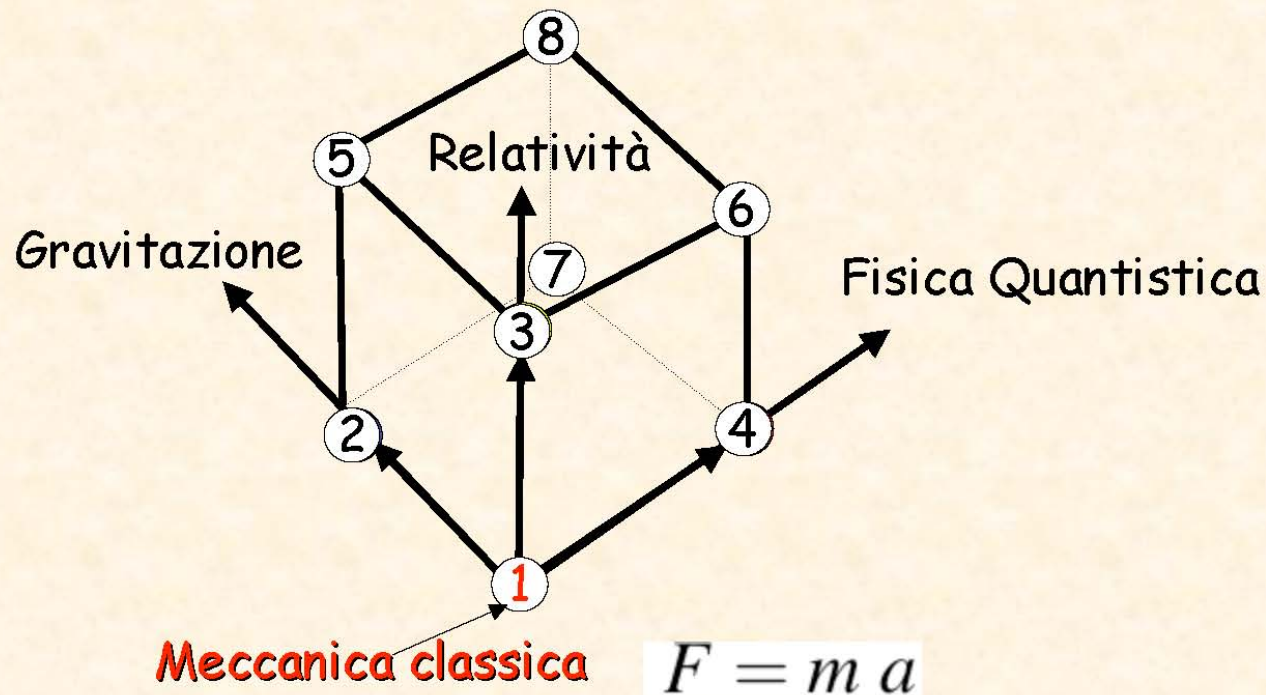


$$L_P = \sqrt{\frac{Gh}{c^3}}$$

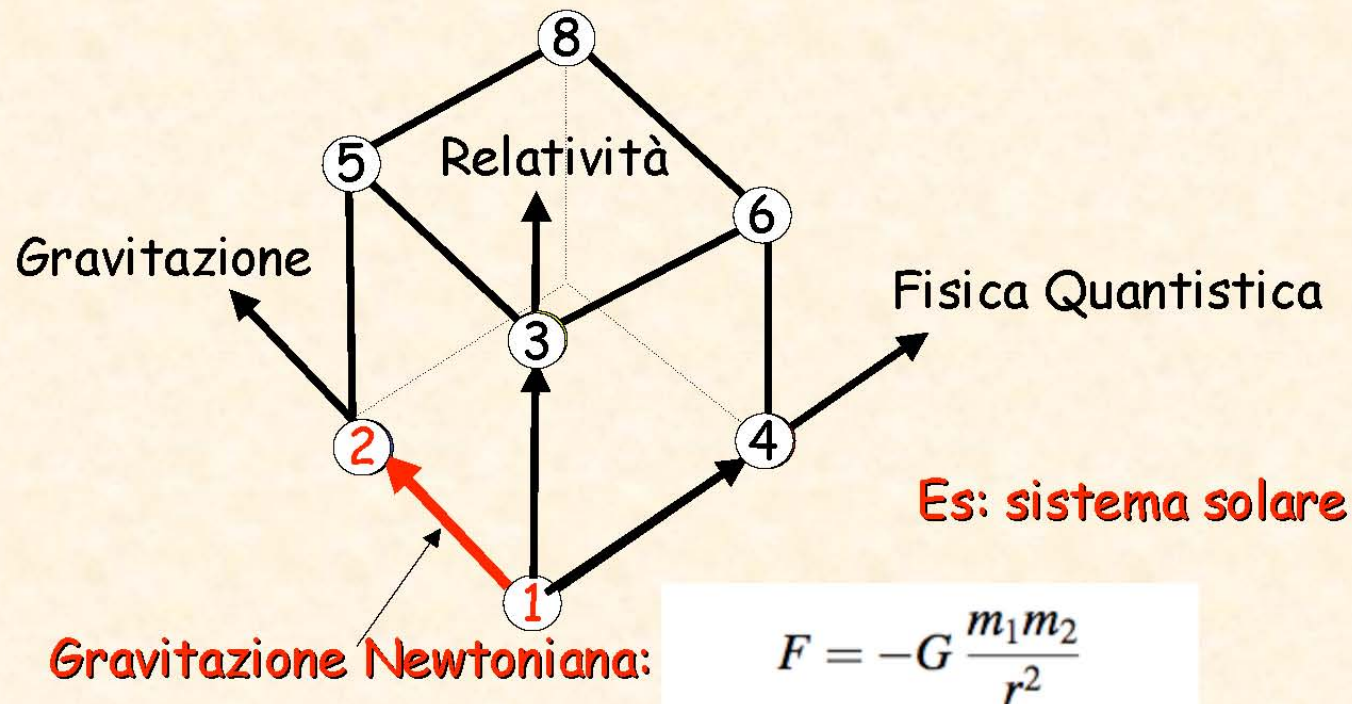
Una «teoria di teorie»



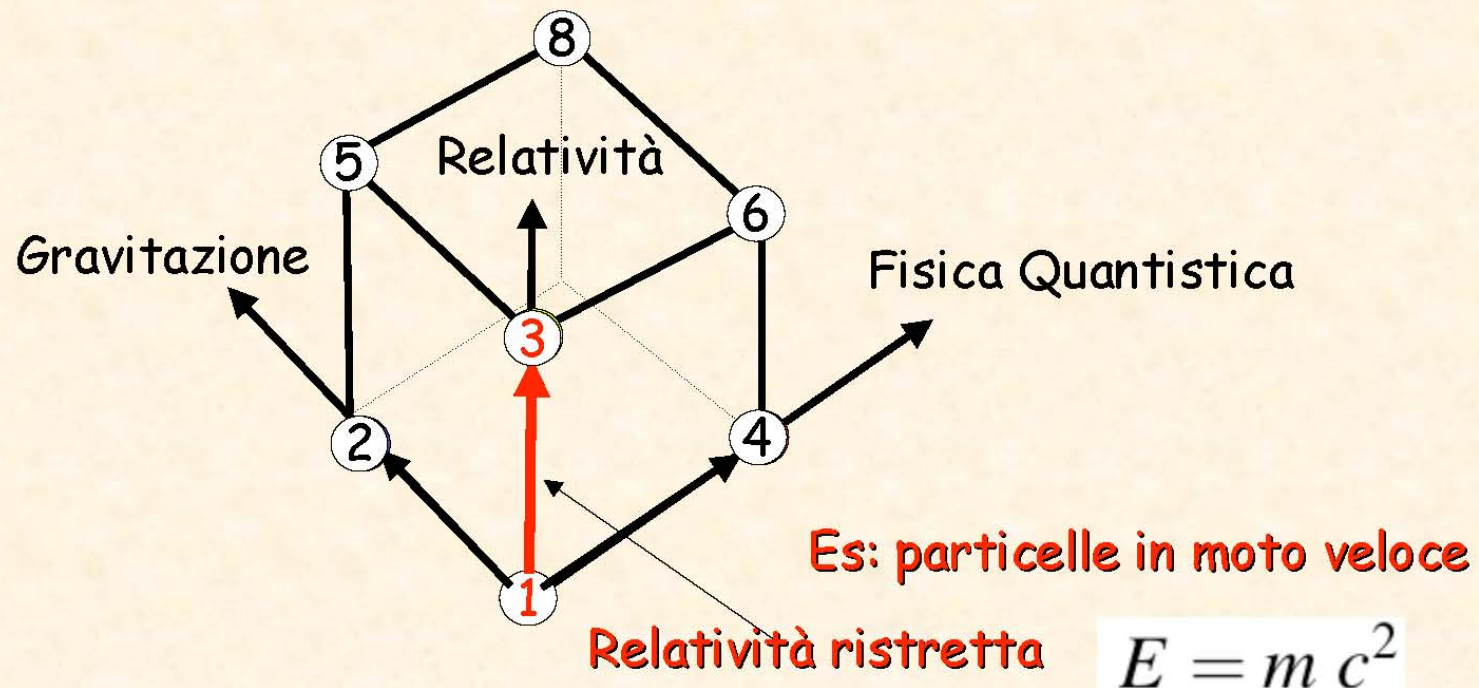
Il vertice più banale



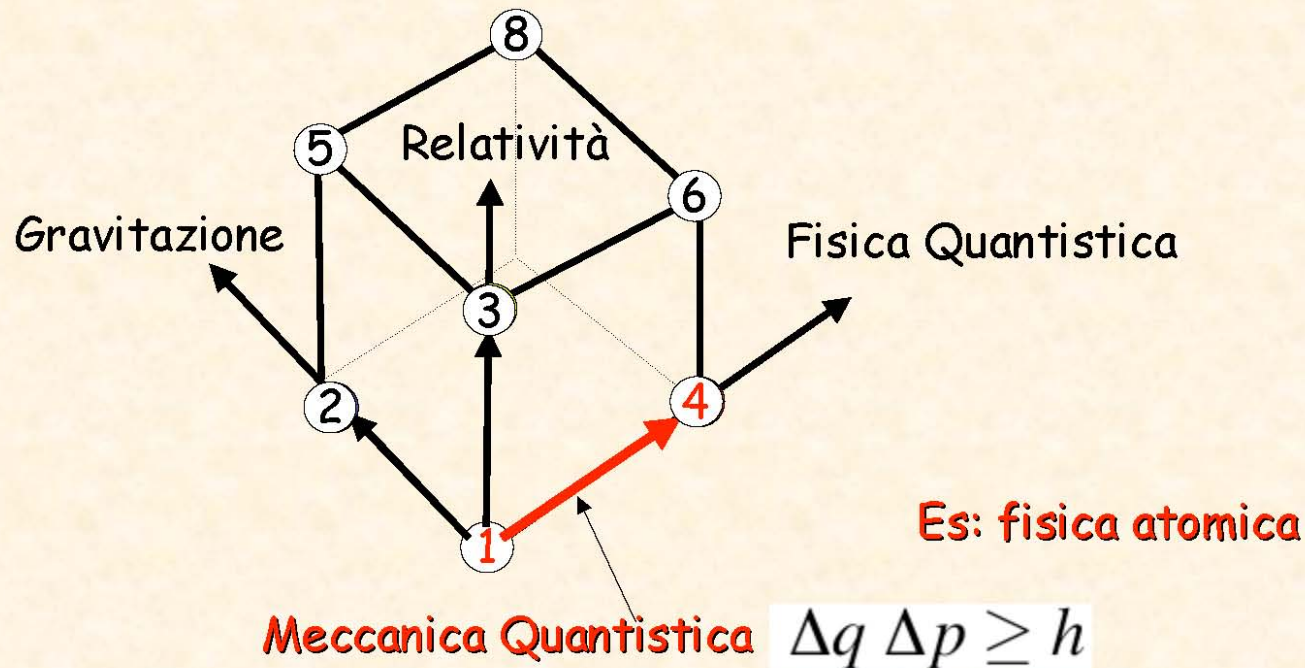
I lati più semplici: I



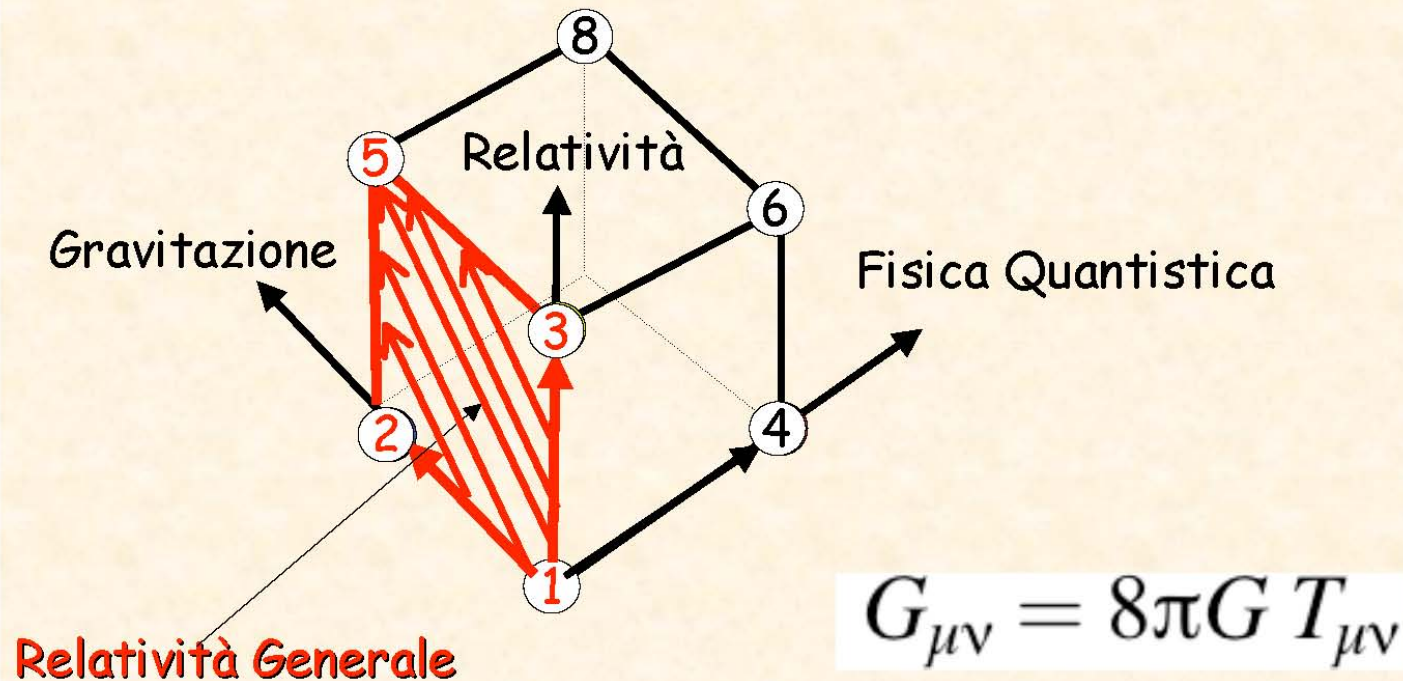
I lati più semplici: II



I lati più semplici: III



Le facce più rilevanti: I



Relatività Generale (RG)

$$GN + RR = RG$$

⇒ «Modello Standard» della gravitazione classica
(sempre meglio confermato tramite verifica delle
correzioni alla teoria Newtoniana; uso nel GPS..)

Predizione di fenomeni nuovi

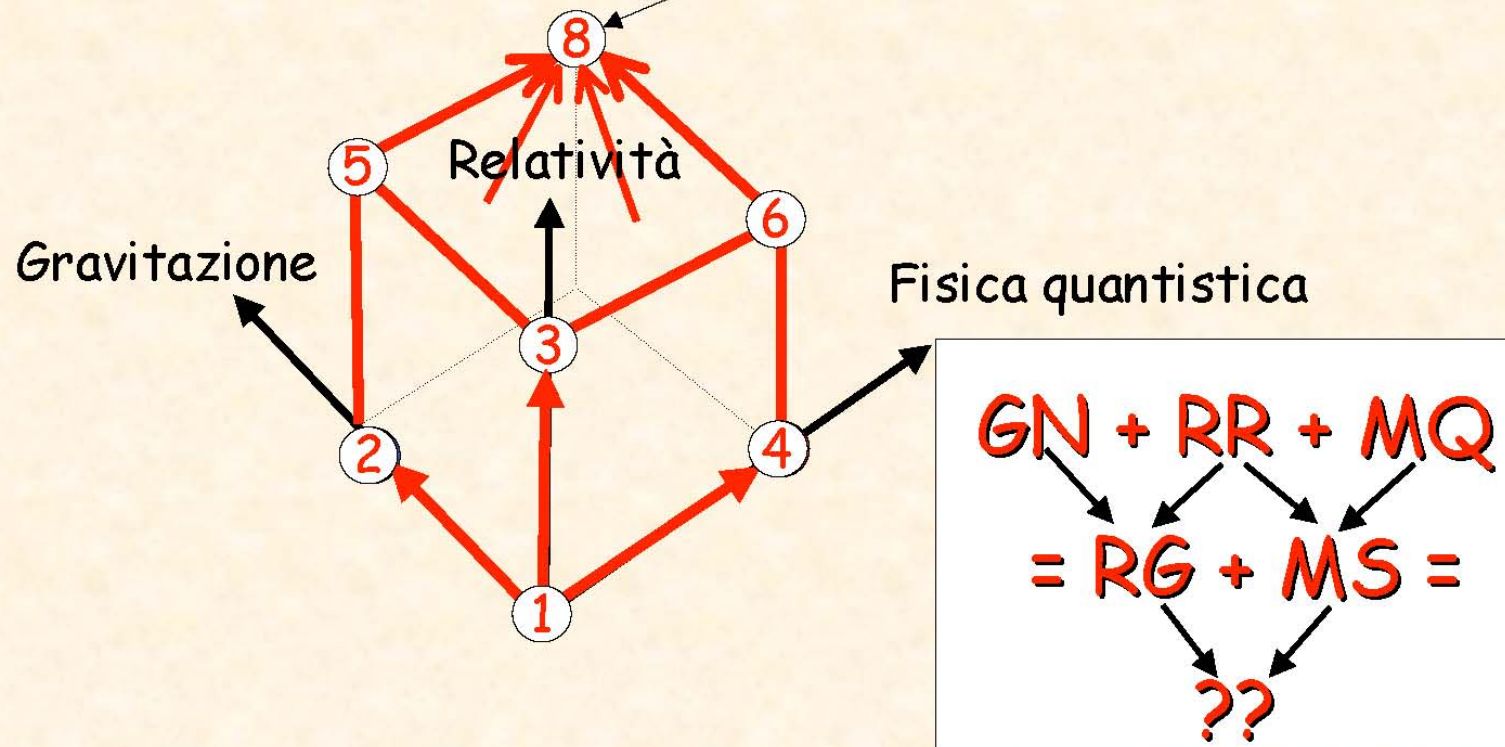
1. **Buchi neri** (evidenza diretta)
2. **Onde gravitazionali** (evidenza indiretta)

$$RR + MQ = MSPE$$

«Modello Standard» delle particelle elementari
(verificato con grande precisione, CERN, Fermilab..)

Il carattere quanto-relativistico del MS si manifesta attraverso la **produzione**, reale e virtuale, di **particelle**. Questi fenomeni inducono «correzioni radiative» che **sono essenziali** per l'accordo con i dati!

Più rimontiamo verso il passato
più ci avviciniamo al vertice no. 8!



Patologie classiche in Relatività Generale

Instabilità gravitazionale e ubiquità delle singolarità

Il carattere attrattivo della forza gravitazionale fa sì che le possibili disomogeneità iniziali tendono a crescere senza limite fino ad arrivare a quella che viene chiamata una **singolarità**. In quel luogo, o a quell'istante, grandezze fisiche quali densità, temperatura, curvatura dello spazio divengono infinite (esempi: interno di un buco nero, big bang). Le singolarità implicano frontiere invalicabili per la nostra descrizione fisica! Ma non è la prima volta che incontriamo una tale situazione..

EP violation expected – To what level?

Fischbach et al. 1995: In a classical framework, if gravity couples anomalously to the energy of neutrino-antineutrino exchange, its contribution to the mass-energy of the nucleus (rigorous calculation) would lead to an EP violation at the level

$$\eta \simeq 10^{-17}$$

Classical and based on rigorous calculation, but achievable only in space...

Damour&Polyakov 1994: Beyond the present framework of physics, predictions based on string theory and the existence of dilaton put an EP violation between $\eta \simeq 10^{-13}$ and $\eta \simeq 10^{-18}$

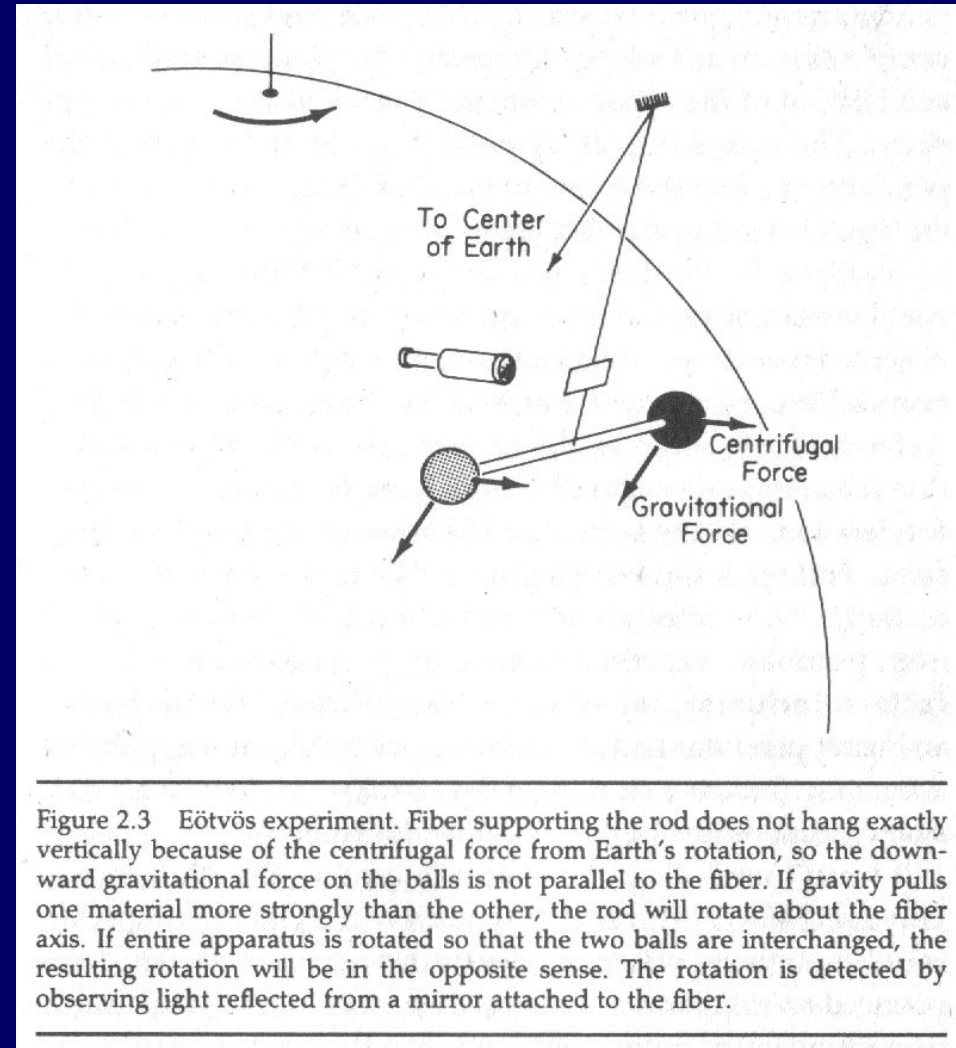
Very interesting, but highly speculative...

Damour, Piazza&Veneziano 2002: With a new scenario for the dilaton, an EP violation could be expected just below $\eta \simeq 10^{-12}$ (for test masses made of Cu, Be or Pt, Ti)

Eotvos' torsion balance

Eötvös (1888-1905/08s): confirms UFF to 1 part in 10^8 by torsion balance tests (improves by 3 orders of magnitude over pendulum tests - **differential instrument!!**)

It was indeed a very subtle idea that a deviation from the proportionality between inertial and gravitational mass should show up as a rotation of the balance (**inherently differential instrument !!**)



Note: Eötvös was not aware of the work by Einstein just in the same years....he had a very sensitive instrument and used it !!!

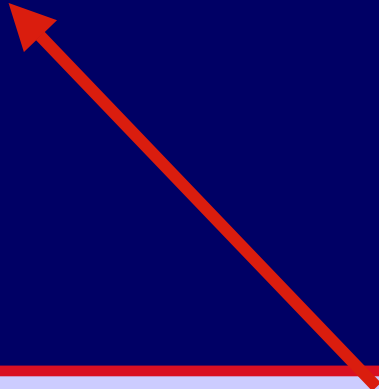
$$\mathbf{a}_{NS}^{\oplus} = \omega_{\oplus}^2 R_{\oplus} \cos \theta \sin \theta \simeq 1.7 \cdot 10^{-2} \text{ ms}^{-2} \quad (\theta = 45^\circ \text{ latitude})$$

Modern torsion balance tests of the Equivalence Principle

“Eöt-Wash” group: EP tests with slowly rotating torsion balance

1994: $\eta(\text{Be}, \text{Cu}) = (-1.9 \pm 2.5) \cdot 10^{-12}$

1999: $\Delta a_{\odot} = (0.3 \pm 5.6) \cdot 10^{-13} \text{ cm s}^{-2}$, $1\sigma = 5.6 \cdot 10^{-13} \text{ cm s}^{-2}$

$$\eta = \frac{\Delta a_{\odot}}{a_{\odot}} \simeq 9.3 \cdot 10^{-13}$$


....the measured value of η is only slightly below
 10^{-12} And it has been so since the 1970s

Driving signal of EP violation: the advantage of space

On Earth:

- If the Earth is the source mass :

$$\mathbf{a}_{NS}^{\oplus} = \omega_{\oplus}^2 R_{\oplus} \cos \theta \sin \theta \simeq 1.7 \cdot 10^{-2} \text{ ms}^{-2} \quad (\theta = 45^\circ \text{ latitude})$$

- If the Sun is the source mass :

$$\mathbf{a}_{NS}^{\odot} = \frac{GM_{\odot}}{d_{\oplus\odot}^3} (r_y \sin \theta - r_z \cos \theta) \quad (\mathbf{a}^{\odot} \simeq 0.6 \cdot 10^{-2} \text{ ms}^{-2})$$

$$\mathbf{a}_{EW}^{\odot} = \frac{GM_{\odot}}{d_{\oplus\odot}^3} r_x \quad (r_x, r_y, r_z : \text{diurnal \& annual motion of } \odot)$$

In space (LEO):

$$\mathbf{a}^{\oplus} = \frac{GM_{\oplus}}{(R_{\oplus} + h)^2} \simeq 8.4 \text{ ms}^{-2} \quad (h \simeq 520 \text{ km})$$

Equivalence Principle tests: what's on

- The best ground tests (with slowly rotating torsion balances, Eöt-Wash group) have reached:

$$\eta \cong 9.3 \cdot 10^{-13}$$

- LLR tests have reached (Williams et al.):

$$\eta \cong 10^{-13}$$

- Proposed and ongoing experiments for EP testing :

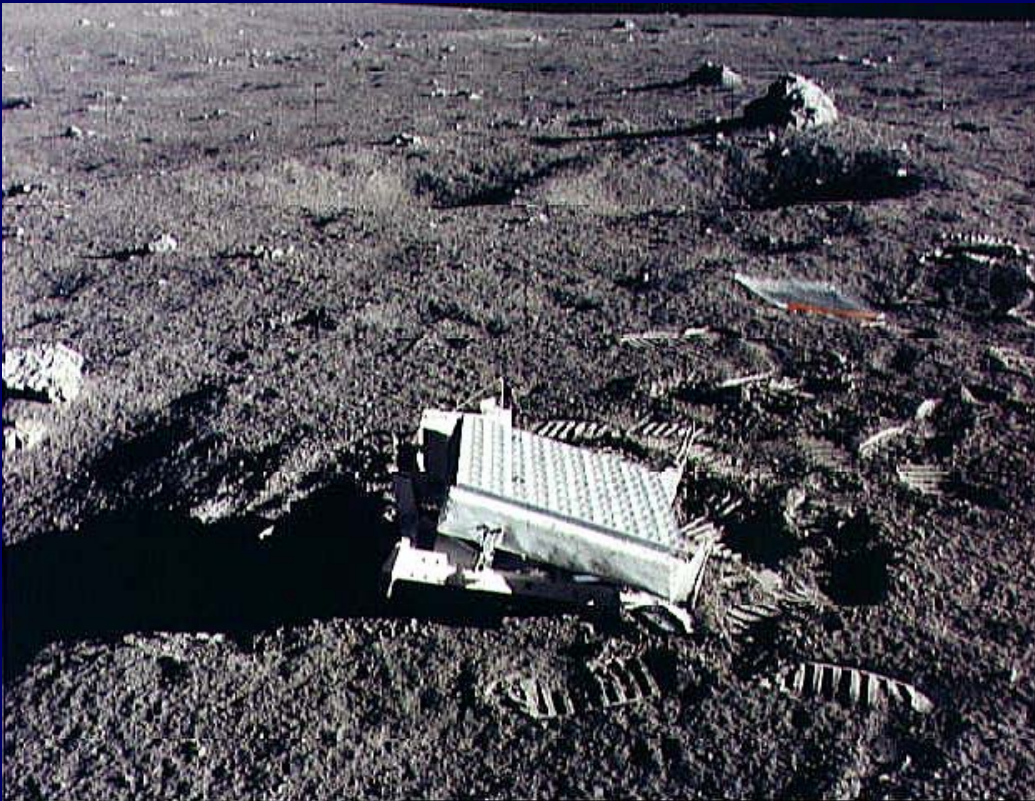
$$\eta \cong 10^{-17}, 10^{-18}$$

GG (I) 250 kg; STEP (USA) 1000 kg-
LEO

$$\eta \cong 10^{-14}, 10^{-15}$$

μ SCOPE (F) 200 kg –LEO; GREaT (I-
USA)

Do the Earth and the Moon fall the same in the gravitational field of the Sun?



The polarization of the lunar orbit towards the Sun is mostly classical (parallactic inequality, because of perturbations due to the Sun; Laplace). The amplitude of polarization would be slightly larger should the Earth and the Moon fall differently towards the Sun because of an EP violation (Nordtvedt effect)

Violation might be due to different composition or different gravitational binding energy, or both...