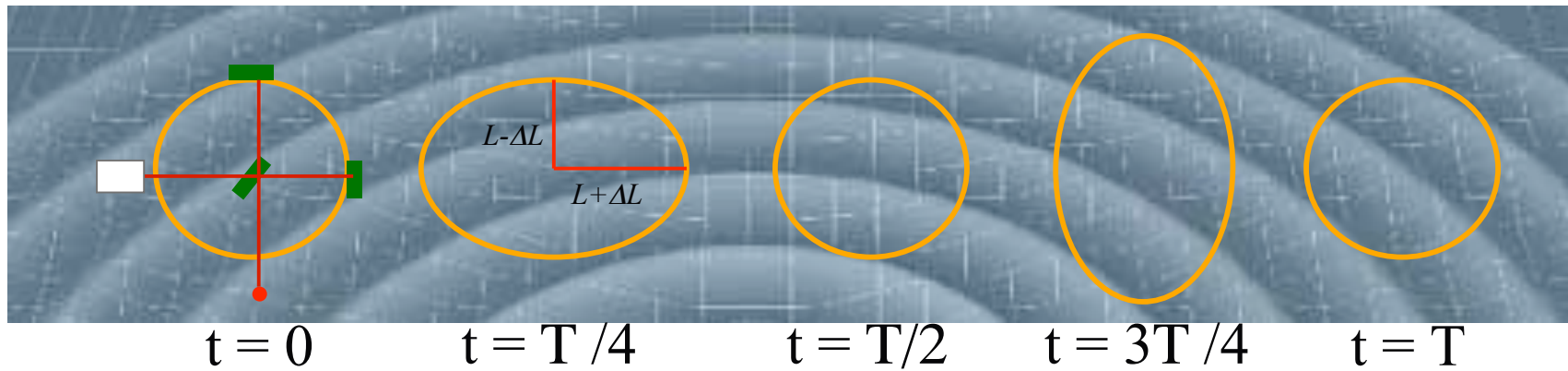


# Virgo e la rivelazione di Onde Gravitazionali

Fulvio Ricci

*Dipartimento di Fisica,  
Università di Roma La Sapienza*

## Rivelazione Interferometrica di OG



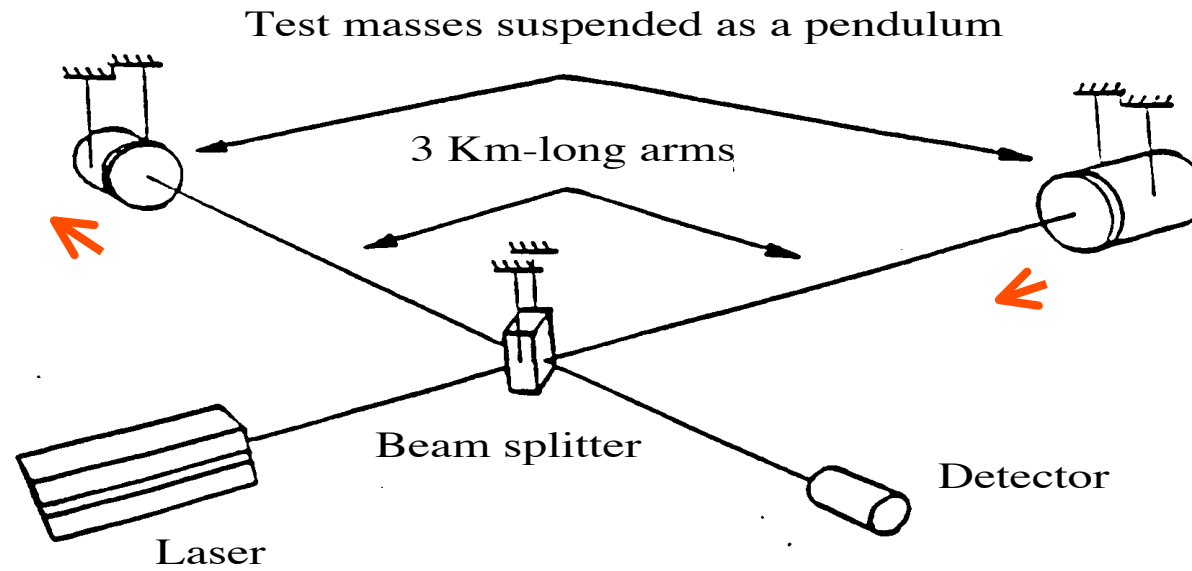
$$\Delta L = \frac{1}{2} h L$$

$$L \sim 10^3 \text{ m}$$

Measure  $\Delta L \sim 10^{-18} \text{ m}$

Target  $h \sim 10^{-21}$   
(NS/NS @15 Mpc)

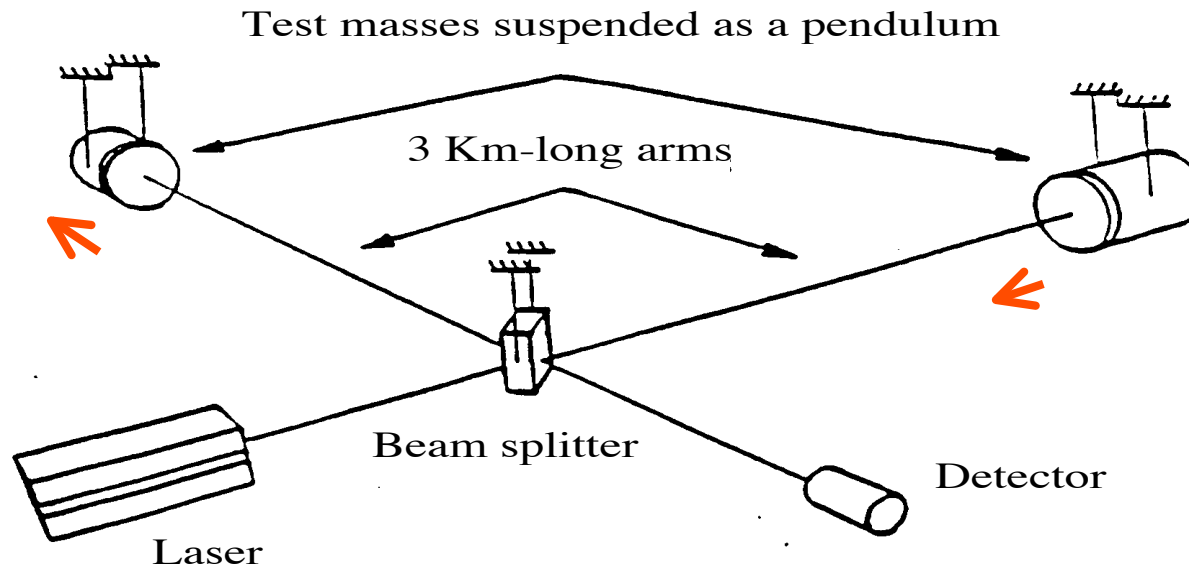
## Un semplice rivelatore



$$\phi_{gw} = \frac{4\pi}{\lambda} L \cdot h$$

$$\phi_{GW} \approx 10^{-11} \text{ rad}$$

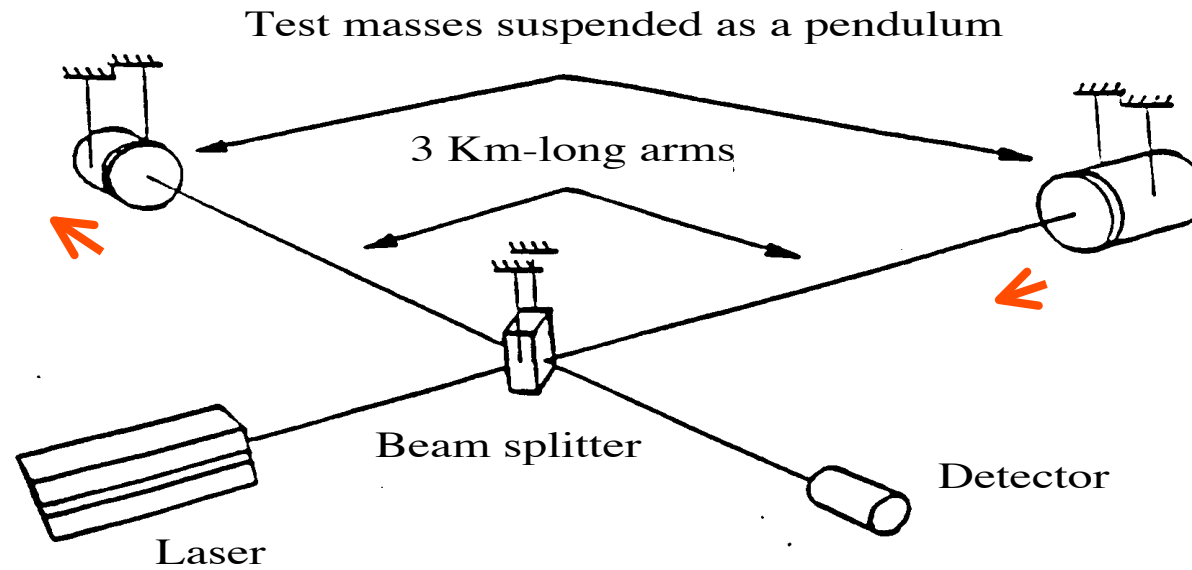
## Un semplice rivelatore



$$\phi_{gw} = \frac{4\pi}{\lambda} L \cdot h$$

*Molte sorgenti di rumore possono intervenire  
nel limitare la sensinbilità richiesta  
(Rumori di spostamento e di fase)*

## Un semplice rivelatore

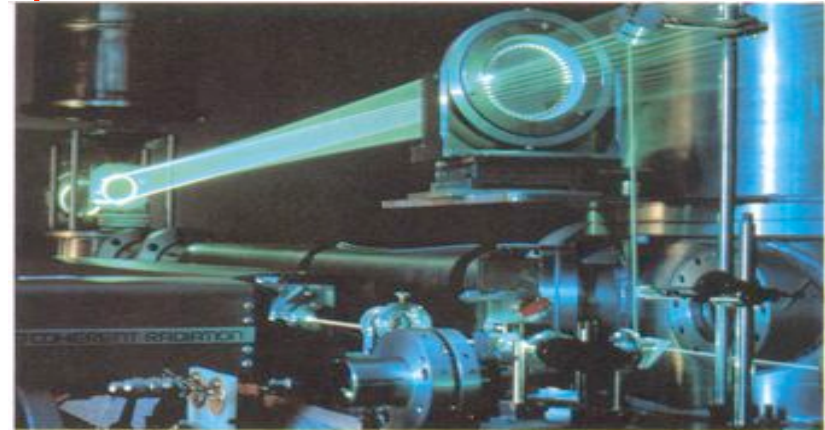
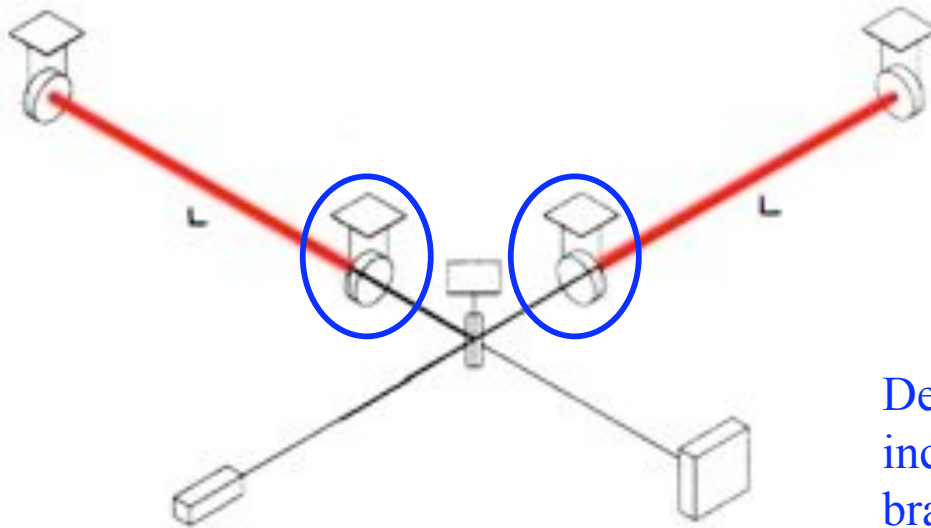


$$P_{out} = P_{in} \cos^2(k_x L_x - k_y L_y) = P_{in} (1 + 2 \cos(k_x L_x - k_y L_y))$$

*Massima variazione di Potenza al variare della differenza di cammino ottico dei due bracci si ha a  $P_{out} = P_{in}/2$*

$$\left( \frac{dP_{out}}{d\Delta L} \right)_{Max} = \frac{2\pi}{\lambda} P_{in}$$

## Delay Lines o cavità Fabry-Perot per aumentare l'effetto



Un prototipo di antenna interferometrica sviluppato al Max Planck Institute di Garching (Germania).

Delay-Lines possono essere utilizzate per incrementare il cammino ottico della luce nei 2 bracci dell'interferometro.

- Cavit  Fabry-Perot

- Storage time

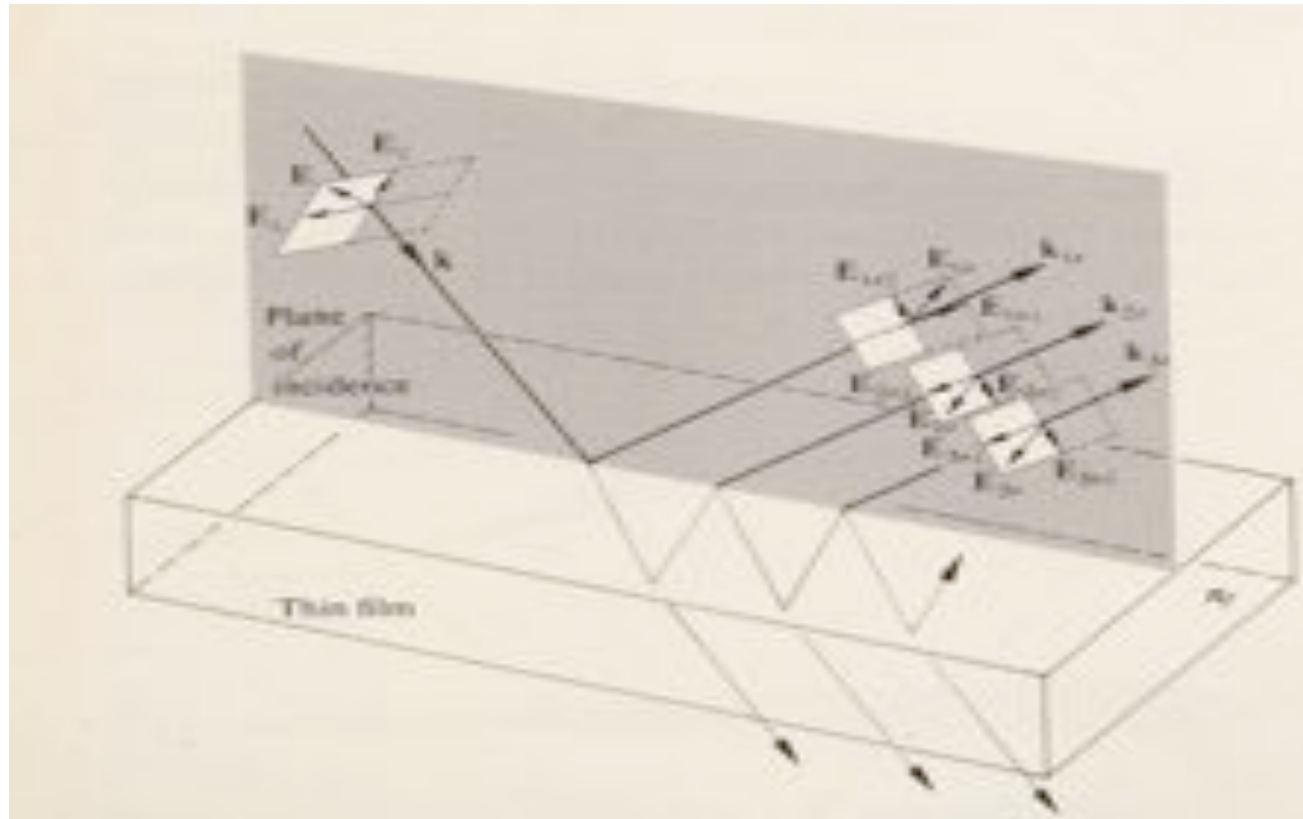
$$\tau_s = \frac{L}{c} \frac{F}{\pi}$$

$$N = \frac{F}{2\pi}$$

$$\tau_s = \frac{2NL}{c}$$

$$\Delta\phi = \frac{4\pi FhL}{c}$$

# Interferenza da strati sottili



**Finesse  $F = [2r / (1 - r^2)]^2$**

**$\delta = (4 \pi / \lambda) n_f L \cos \theta$**

**Per l'Interferometro**

**Gravitazionale**

**$\theta = 0$  e  $n_f = 1$**

**$\delta / 2 = (2 \pi / \lambda) L$**

**Intensità  
riflessa**

**Intensità  
incidente**

**Intensità  
trasmessa**

**Intensità  
incidente**

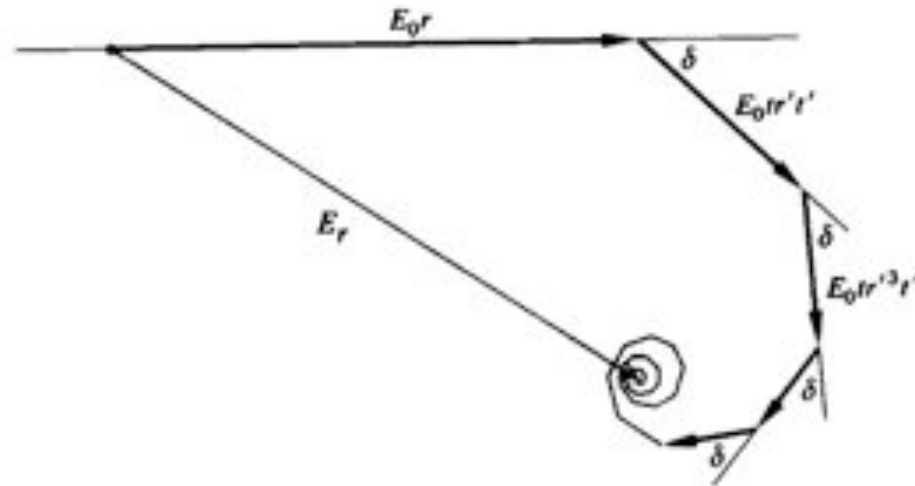
$$\frac{I_r}{I_i} = \frac{F \sin^2 (\delta/2)}{1 + F \sin^2 (\delta/2)}$$

$$\frac{I_t}{I_i} = \frac{1}{1 + F \sin^2 (\delta/2)}$$

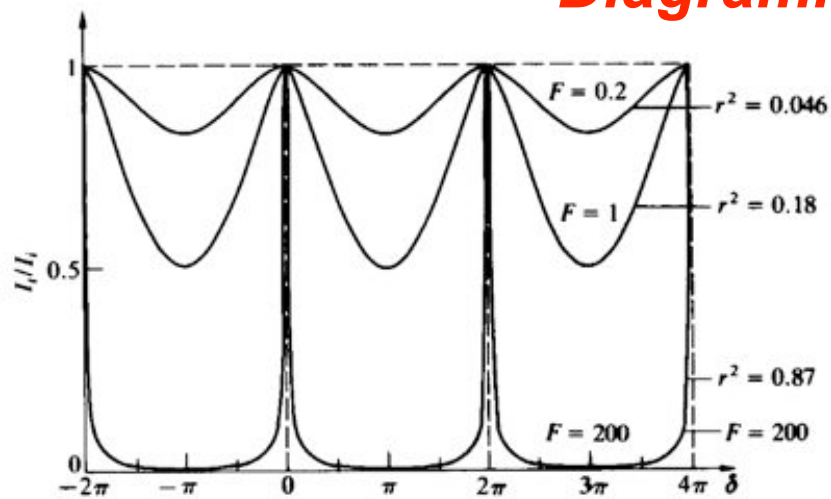
**1- Funzione  
di Airy**

**Funzione di  
Airy**

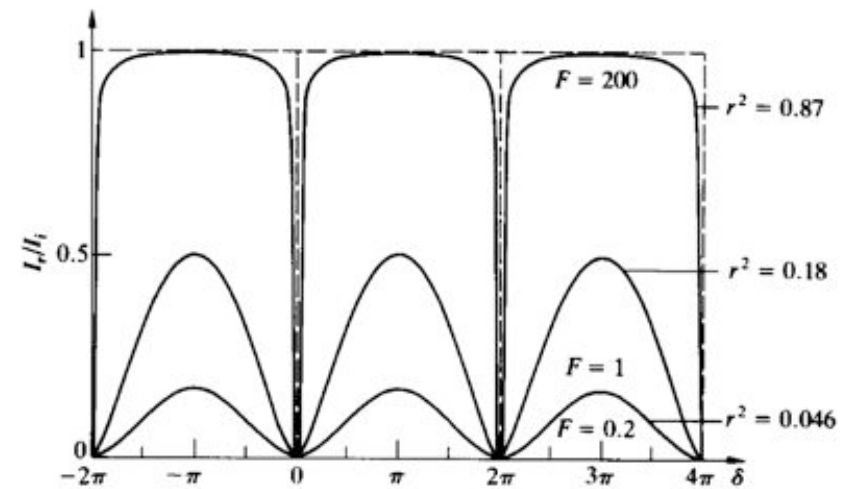
# Interferenza da strati sottili



**Diagramma dei Fasori**

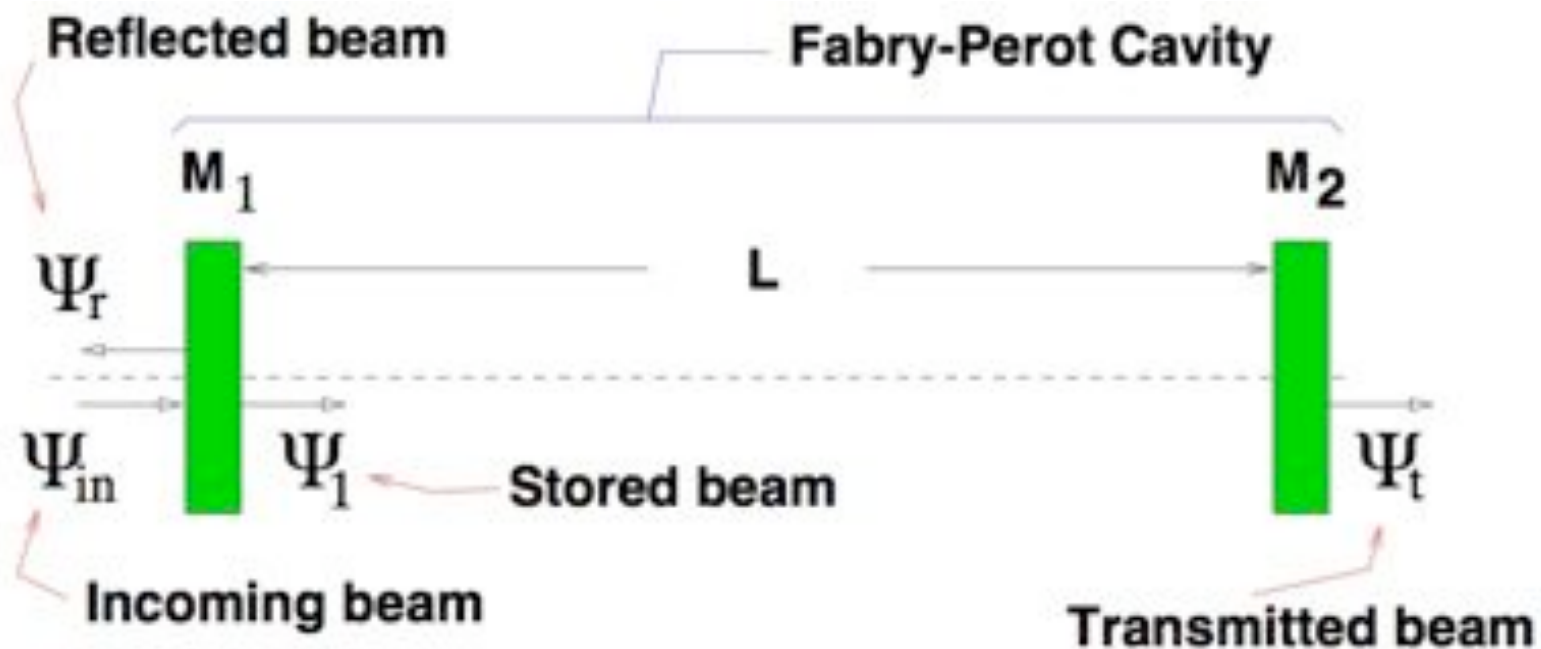


**Funzione di Airy**



**1 - Funzione di Airy**

# Cavità Fabry-Perot: la trattazione estesa nel caso di due specchi paralleli con $r_i^2 + t_i^2 = 1$



## Trattazione delle cavità FP (I)

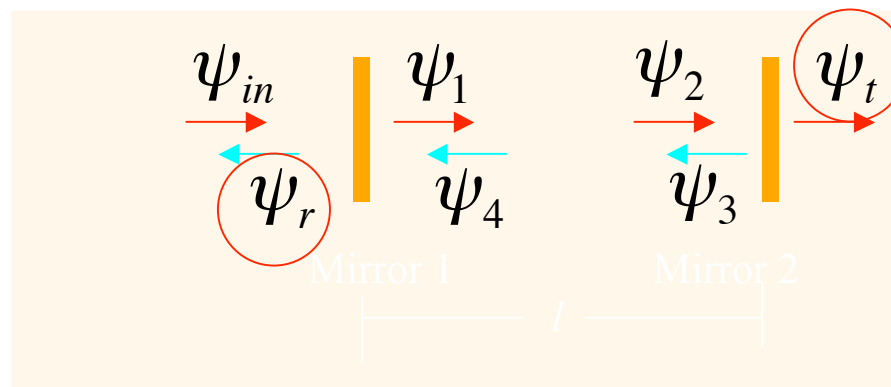
$$\psi_{in} = K e^{i\chi}$$

$$\psi_1 = \text{unknown}$$

$$\psi_2 = e^{-ikl} \psi_1$$

$$\psi_3 = ir_2 \psi_2$$

$$\psi_4 = e^{-ikl} \psi_3$$



**Stato stazionario:**  $\psi_1 = t_1 \psi_{in} + ir_1 \psi_4$

$$\psi_1 = \frac{t_1}{1 + r_1 r_2 e^{-2ikl}} \psi_{in}$$

$$\psi_4 = \frac{ir_2 t_1}{1 + r_1 r_2 e^{-2ikl}} \psi_{in}$$

**Onda Riflessa:**  $\psi_r = ir_1 \psi_{in} + t_1 \psi_4 = i \frac{r_1 + r_2 e^{-2ikl}}{1 + r_1 r_2 e^{-2ikl}} \psi_{in}$

**Onda Transmessa:**  $\psi_t = t_2 \psi_2 = \frac{t_1 t_2 e^{-ikl}}{1 + r_1 r_2 e^{-2ikl}} \psi_{in}$

## Trattazione delle cavità FP (II)

Ampiezza dell'onda:

$$|\psi_r|^2 = |A_r|^2$$

$$|\psi_t|^2 = |A_t|^2$$

$$A_r = \sqrt{\frac{r_1^2 r_2^2 + 2r_1 r_2 \cos 2kl}{1 + r_1^2 r_2^2 + 2r_1 r_2 \cos 2kl}}$$

$$A_t = \sqrt{\frac{1}{1 + r_1^2 r_2^2 + 2r_1 r_2 \cos 2kl}} t_1 t_2$$

$$L = (2m + 1) \frac{\lambda}{4}$$

se

$$L = m\lambda$$

$$A_r = \min = \frac{r_2 - r_1}{1 - r_1 r_2}; A_t = \max = \frac{t_1 t_2}{1 - r_1 r_2}$$

$$A_r = \max = \frac{r_2 + r_1}{1 + r_1 r_2}; A_t = \min = \frac{t_1 t_2}{1 + r_1 r_2}$$

## Cavità Fabry-Perot:

$$|\psi_1(\phi)|^2 = G \frac{1}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2(\phi/2)}$$

where  $G$  is the gain in power defined as the ratio between the stored and incident power in condition of resonance:

$$G \equiv \frac{t_1^2}{(1 - r_1 r_2)^2}$$

Often, the mirror reflectivities not only satisfy the condition  $1 - r_1 r_2 \ll 1$ , but also the condition  $1 - r_1 \gg 1 - r_2$ . In this case, it can be shown that the relationship between the gain  $G$  and the finesse  $\mathcal{F}$  is

$$G \simeq \frac{2\mathcal{F}}{\pi}$$

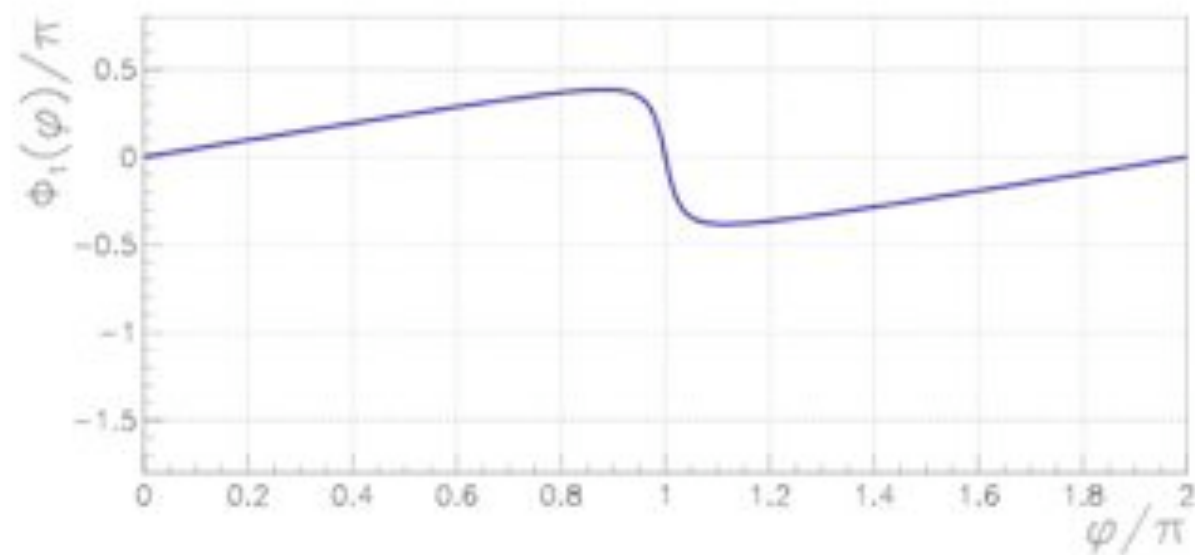
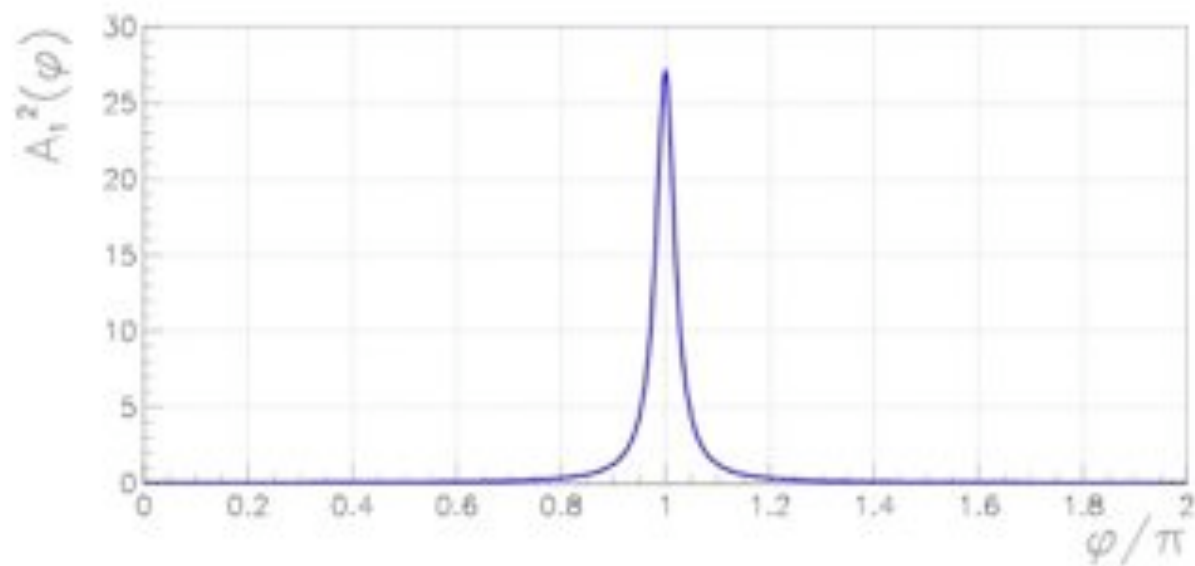
the phase reflectivity of a Fabry-Perot plays a central role in interferometric detection. To first order approximation, it is possible to show that the phase change induced by a perturbation of the cavity length  $\delta L$  around resonance is

$$\phi_R(L + \delta L) - \phi_R(L) \simeq 2kG\delta L$$

$$\text{and} \quad k = \frac{2\pi}{\lambda}$$

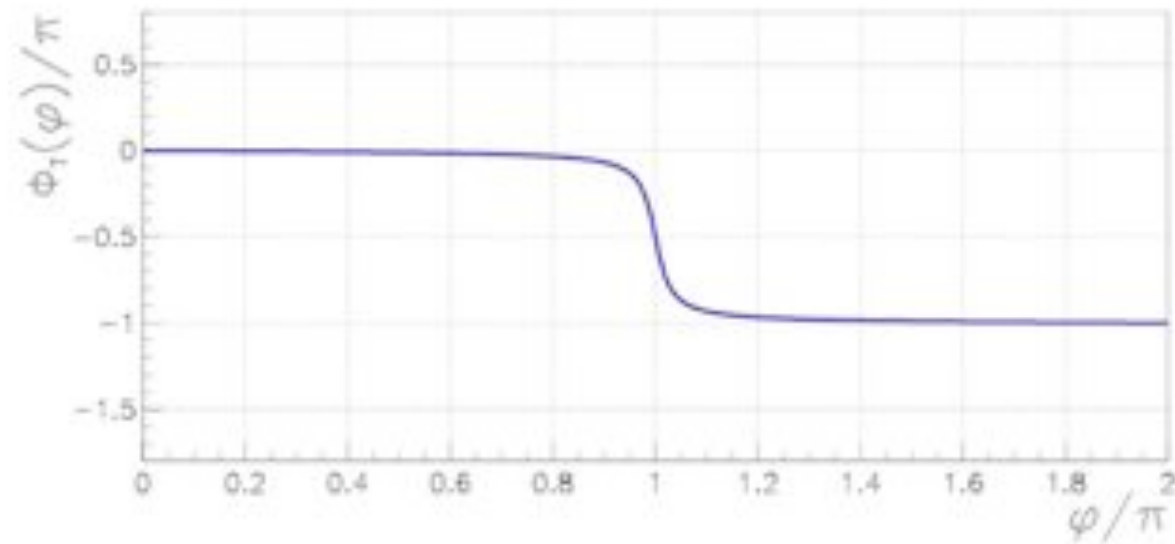
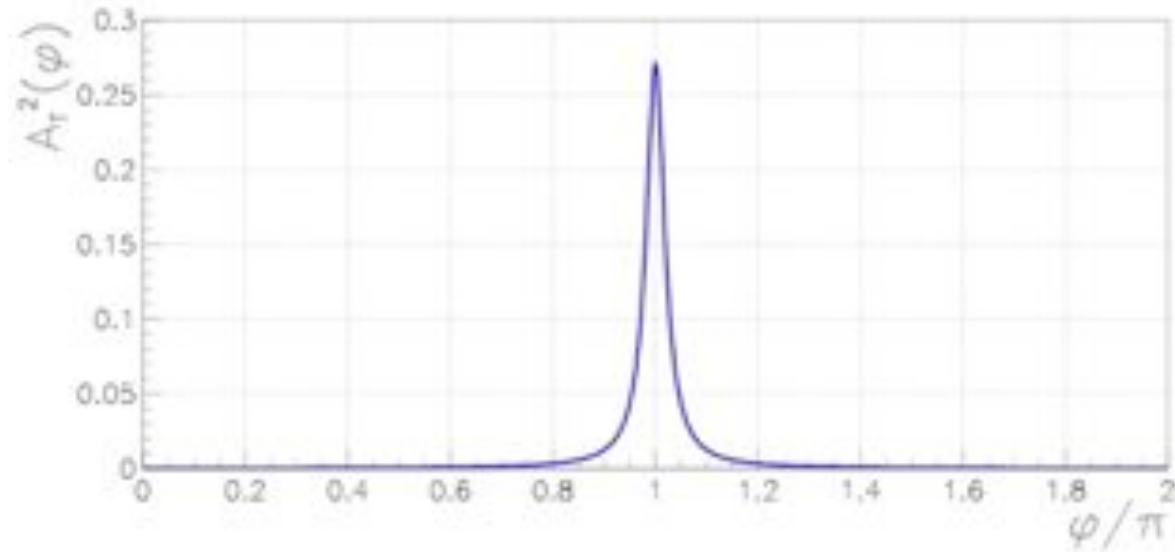
# Cavità Fabry-Perot: luce intrappolata

$r_1=0.88$   
 $r_2=0.99$



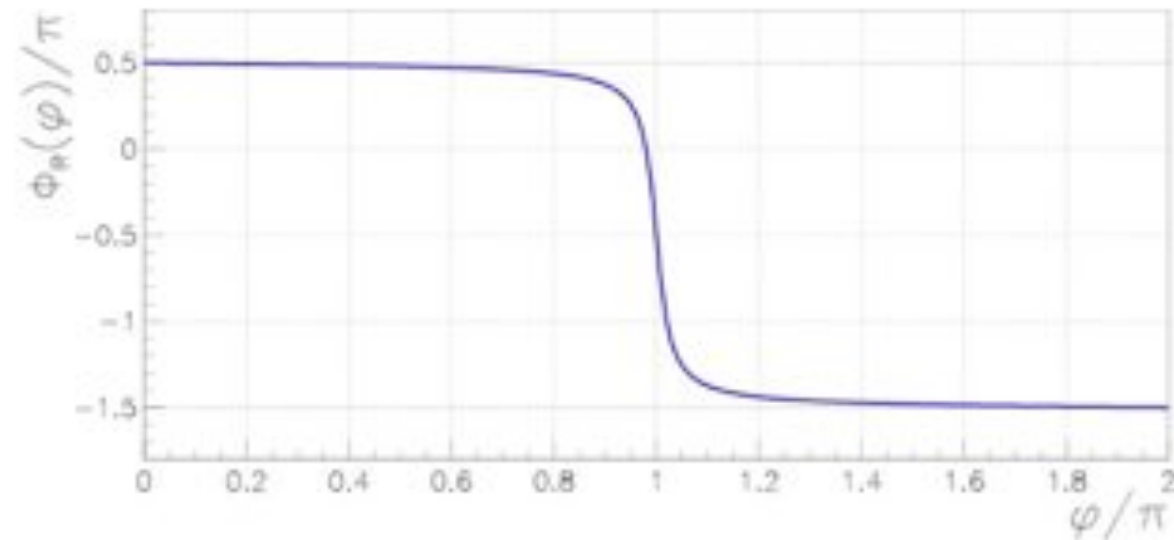
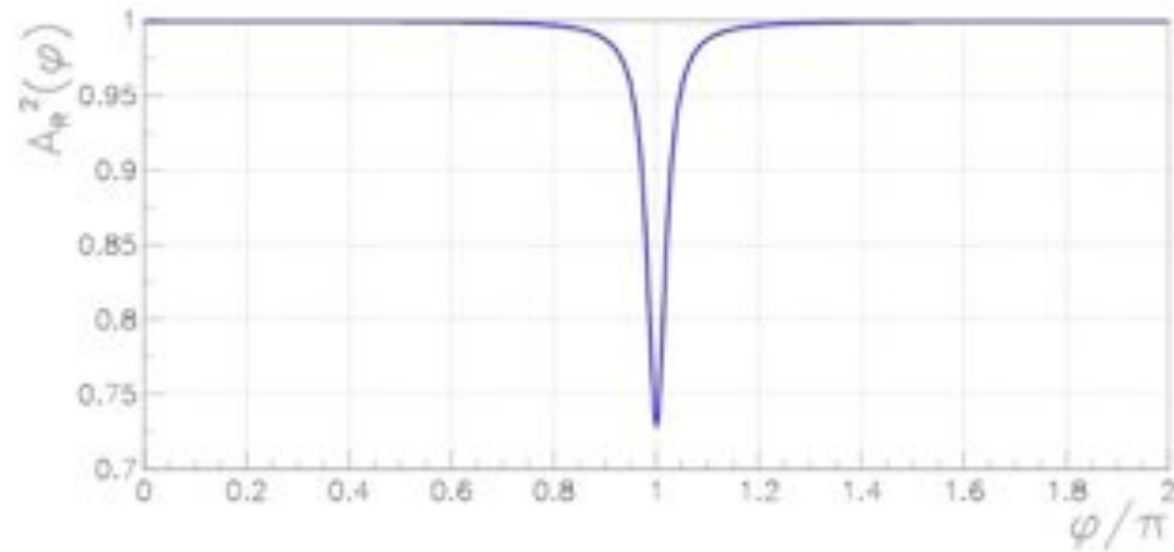
# Cavità Fabry-Perot: luce trasmessa

$$r_1 = 0.88$$
$$r_2 = 0.99$$



# Cavità Fabry-Perot: luce riflessa

$$r_1 = 0.88$$
$$r_2 = 0.99$$



## Trattazione delle cavità FP (III)

*Finesse:*

$$F = \frac{FSR}{FWHM} \approx \frac{\pi \sqrt{r_1 r_2}}{1 - r_1 r_2}$$

*full width @ half max*

*Free Spectral Range:*

$$FSR = \frac{c}{2L}$$

*Round trip number:*

$$N = \frac{F}{2\pi}$$

*Cavity cut-off:*

$$\omega_c = 2\pi \frac{FWHM}{2} = \pi \frac{c}{2FL} = \frac{1}{\tau_s}$$

*Storage time*

*Recycling factor :*

$$FI = \left| \frac{\psi_{3;L=(2m+1)\frac{\lambda}{4}}}{\psi_{in}} \right| = \frac{t_1^2}{(1 - r_1 r_2)^2}$$

## Trattazione delle cavità FP (IV)

*Lunghezza Ottica della cavità  
(si noti che dipende dalla frequenza)*

$$L_{opt} = FL \frac{2}{\pi} \frac{1}{\sqrt{1 + (\omega / \omega_c)^2}}$$

**In un interferometro con cavità FP il rumore originariamente a spettro bianco nel sistema (ad esempio lo shot noise) risulta deformato e dipende dalla frequenza di cut-off  $\omega_c$ .**

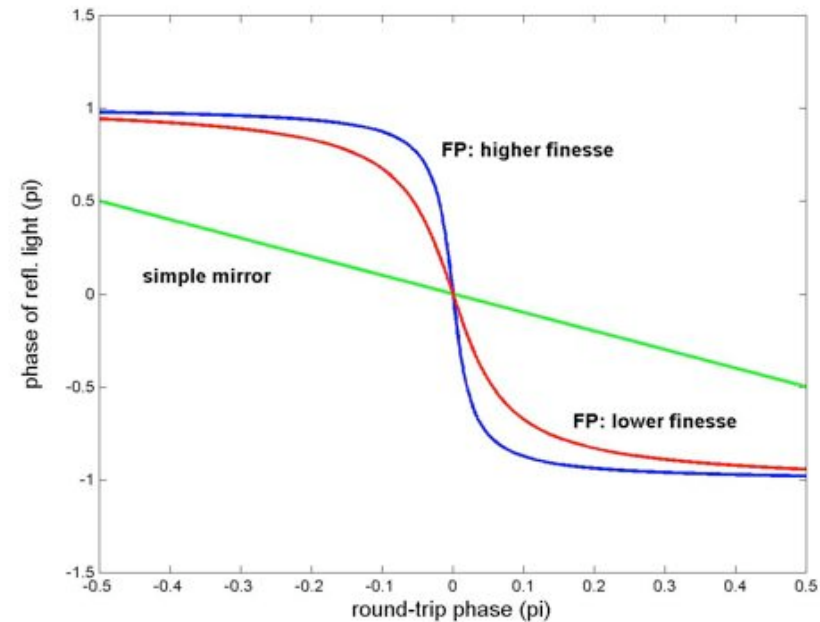
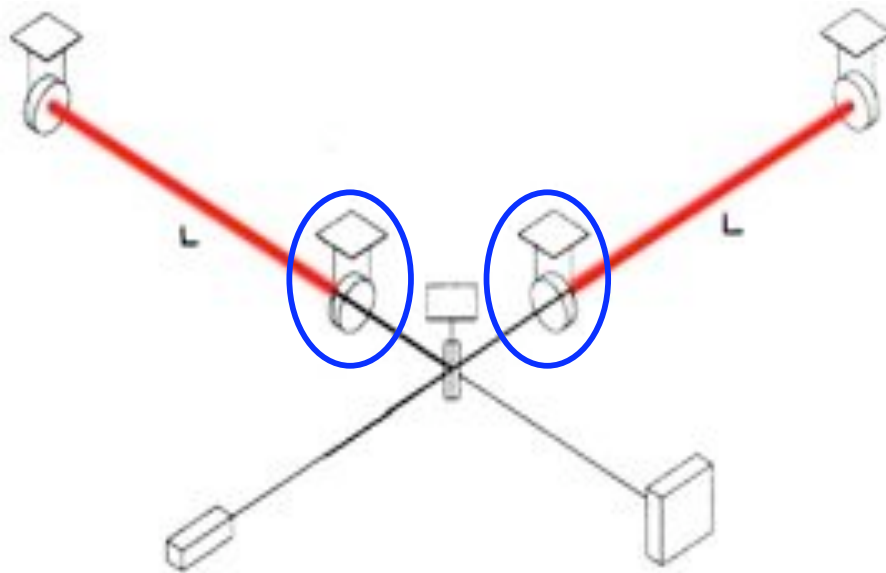
### FP-+Michelson ITF

Il formalismo per calcolare il campo e.m. d'uscita del Michelson+FP è analogo a quello usato nel caso del Michelson semplice, in cui però la riflettività degli specchi è stata sostituita con quella delle cavità FP:

$$r_1 = A^{(1)}_r \quad r_2 = A^{(2)}_r$$



## FP+Michelson ITF, amplificazione del segnale ( sintesi)



$$\frac{d\varphi_r}{dL} \approx -2k \frac{2F}{\pi}$$

- Cavità Fabry-Perot : amplifica la trasduzione lunghezza -fase
- Finesse alta  $\rightarrow d\phi/dL$  più grande
- *Aspetto negativo: l'interferometro funziona solo se le FP sono in risonanza*

# Cos'è il rumore

*Il rumore si manifesta nella variazione casuale di grandezze fisiche*

*Non esiste una legge oraria che lo determini → Caratterizzabile solo attraverso variabili statistiche*

*Un processo si dice **STAZIONARIO** se la misura delle sue proprietà statistiche è indipendente dall'epoca in cui essa viene effettuata.*

$$< x > \quad \text{Media} \qquad \text{Varianza } < (x - < x >)^2 >$$

$$S_x(f) = \lim_{T \rightarrow \infty} \frac{2}{T} |X_T(f)|^2 \quad \text{Potenza Spettrale} \quad \longrightarrow \quad \text{Densità Spettrale}$$

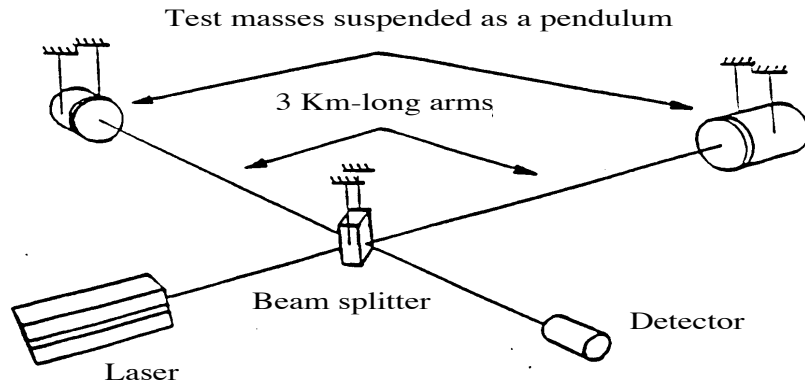
$[X^2 \text{ Hz}^{-1}]$                        $[X \text{ Hz}^{-1/2}]$

$$C_x(t) = \langle X(t) \cdot X(t + \tau) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} X(t) \cdot X(t + \tau) dt \quad \text{Auto Correlazione}$$

$$< x^2 > = \int_0^{\infty} S_x(f) df$$

$$S_x(f) = \int_0^{\infty} C_x(\tau) \cos(2\pi f \tau) d\tau$$

## Lo Shot-Noise



$$N = \frac{P_{in} \tau}{2\hbar\omega} = \frac{P_{in} \tau \lambda}{4\pi\hbar c}$$

*Numero di fotoni al fotodiodo in un tempo  $\tau$*

$$\frac{\sigma_N}{N} = \frac{1}{\sqrt{N}} = \sqrt{\frac{4\pi\hbar c}{\lambda P_{in} \tau}}$$

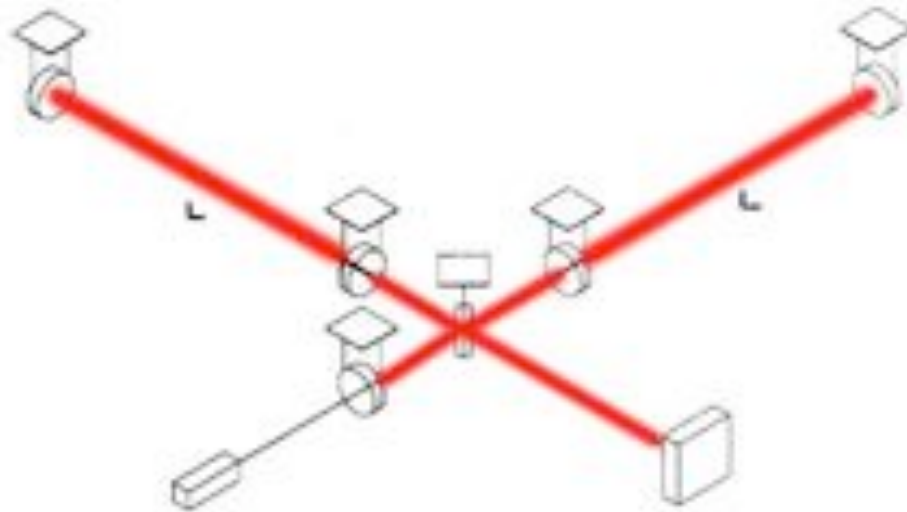
*La fluttuazione di potenza si manifesta come un errore sulla misura del segnale*

$$\frac{\sigma_N}{N} = \frac{\sigma_\phi}{\phi} = \sqrt{\frac{4\pi \hbar c}{\lambda P_{in} \tau}} \frac{\lambda}{4\pi L h} = \frac{\sigma_h}{h} \rightarrow \rightarrow \boxed{\sigma_h = \frac{1}{L} \sqrt{\frac{\lambda \hbar c}{4\pi \underbrace{P_{in}}_{\text{Shot-Noise}} \tau}}}$$

# L'interferometro riciclato

## SPECIFICHE

- 1) Si usano laser ad alta potenza (20 W)
- 2) La luce viene “ricircolata” nell'interferometro per aumentare la potenza del fascio



20 W  $\rightarrow$  1 kW

## Fluttuazioni della pressione di radiazione

Si riduce lo Shot-Noise ma si incrementano le fluttuazioni della  
pressione di Radiazione sugli specchi

$$\tilde{h}_{rp} = \frac{2}{m\omega^2 L} \sqrt{\frac{2\pi\hbar P}{c\lambda}}$$

$$\tilde{h}(f) = \sqrt{\tilde{h}_{shot}^2(f) + \tilde{h}_{rp}^2(f)}$$



$$P_{opt} = \pi c \lambda m f^2$$

*Potenza ottimale alla frequenza  $f$*

$$F = P/c$$

$$\tilde{h}_{QL}(f) = \frac{1}{\pi f L} \sqrt{\frac{\hbar}{M}}$$

**I bracci devono essere lunghi  
in ogni caso**

*Quantum Limit*

## Fluttuazioni della pressione di radiazione

Si riduce lo Shot-Noise ma si incrementano le fluttuazioni della  
pressione di Radiazione sugli specchi

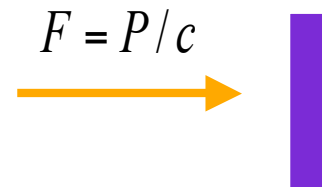
$$\tilde{h}_{rp} = \frac{2}{m\omega^2 L} \sqrt{\frac{2\pi\hbar P}{c\lambda}}$$

$$\tilde{h}(f) = \sqrt{\tilde{h}_{shot}^2(f) + \tilde{h}_{rp}^2(f)}$$



$$P_{opt} = \pi c \lambda m f^2$$

*Potenza ottimale alla frequenza  $f$*



$$\begin{array}{l} m = 10 \text{ kg} \\ f = 100 \text{ Hz} \\ \lambda = 0.545 \text{ }\mu\text{m} \end{array} \longrightarrow P_{opt} = 0.5 \text{ MW}$$

## Std readout: meter/mechanics: back-action

Densità spettrale di rumore di spostamento  
dovuto alla Pressione di Radiazione sugli  
specchi sospesi

$$\langle \tilde{x}_{RP} \rangle^2 = \frac{1}{M \cdot 2\pi\nu^2} \frac{\tilde{P}_{shot}}{c}$$

$$\tilde{h}_{meter} = \frac{1}{L_{FP}} \sqrt{\frac{c_{shot}}{F^2 P} + \frac{c_{RP}}{(M\nu^2)^2} F^2 P}$$

$M$  = massa sospesa dello specchio;  $c_{shot} = c_{shot}(const_{shot})$ ;  $c_{RP} = c_{RP}(const_{RP})$

$F$  e  $P$  non possono crescere  
senza indurre un'azione  
meccanica

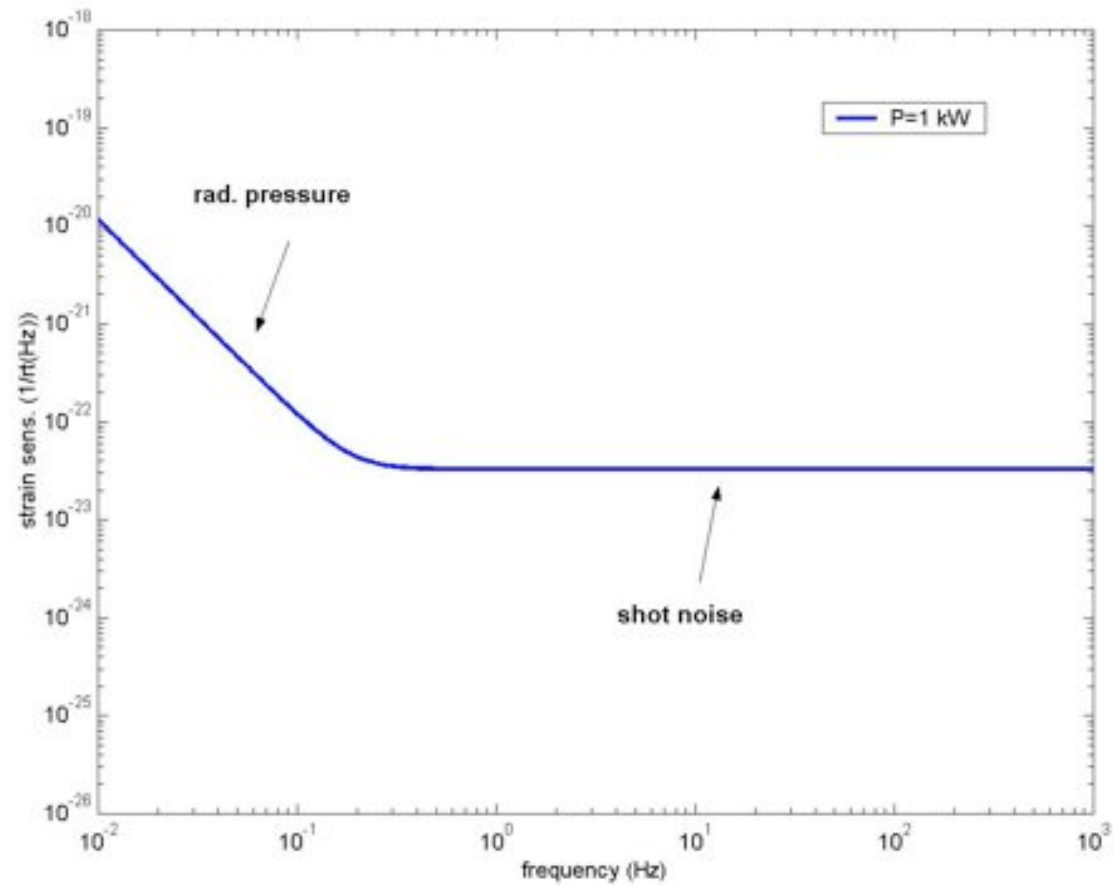
$M$  può esser scelto in modo  
da ridurre l'effetto

$$3 \cdot 10^{-21} / \sqrt{Hz} @ 10Hz \geq \tilde{h}_{TN} \geq \tilde{h}_{meter}$$

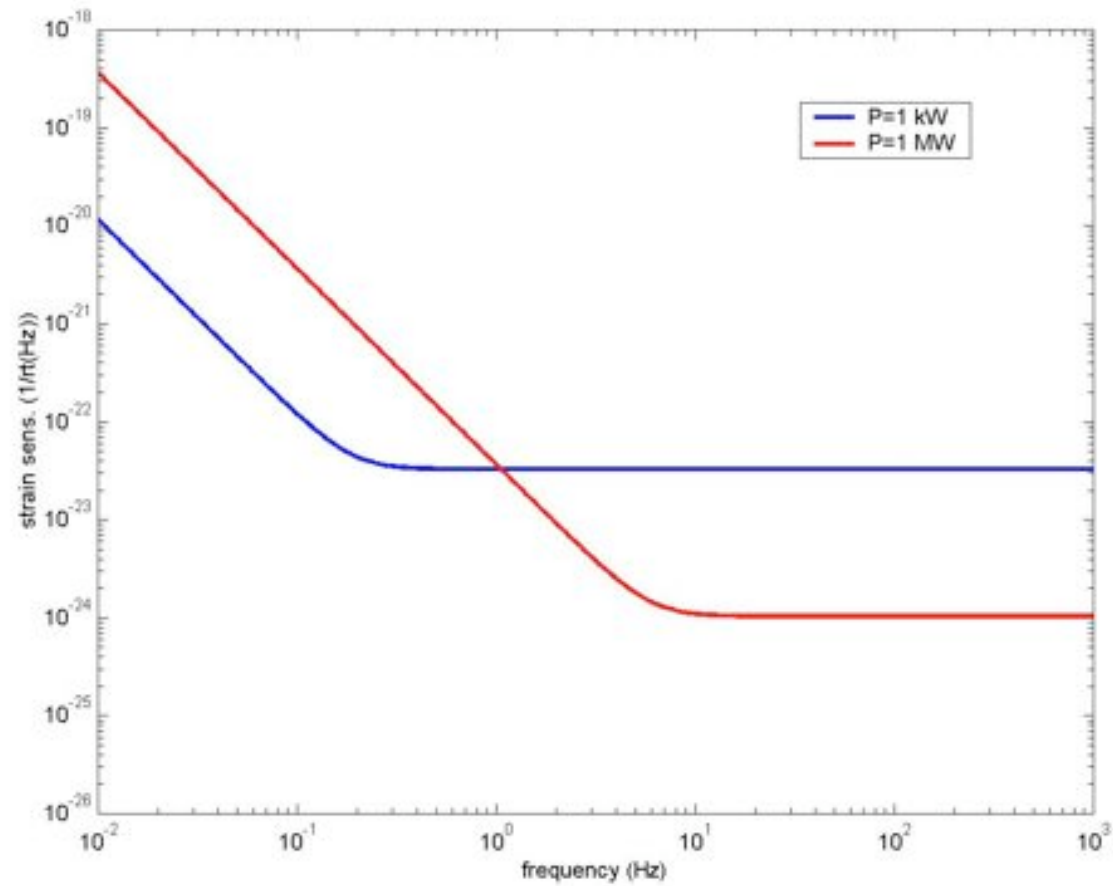
SQL è oggi ancora lontano ma, se si riesce ad alzare  $P$  ed il  
thermal noise è basso come è possibile ottimizzare...

thermal  
noise

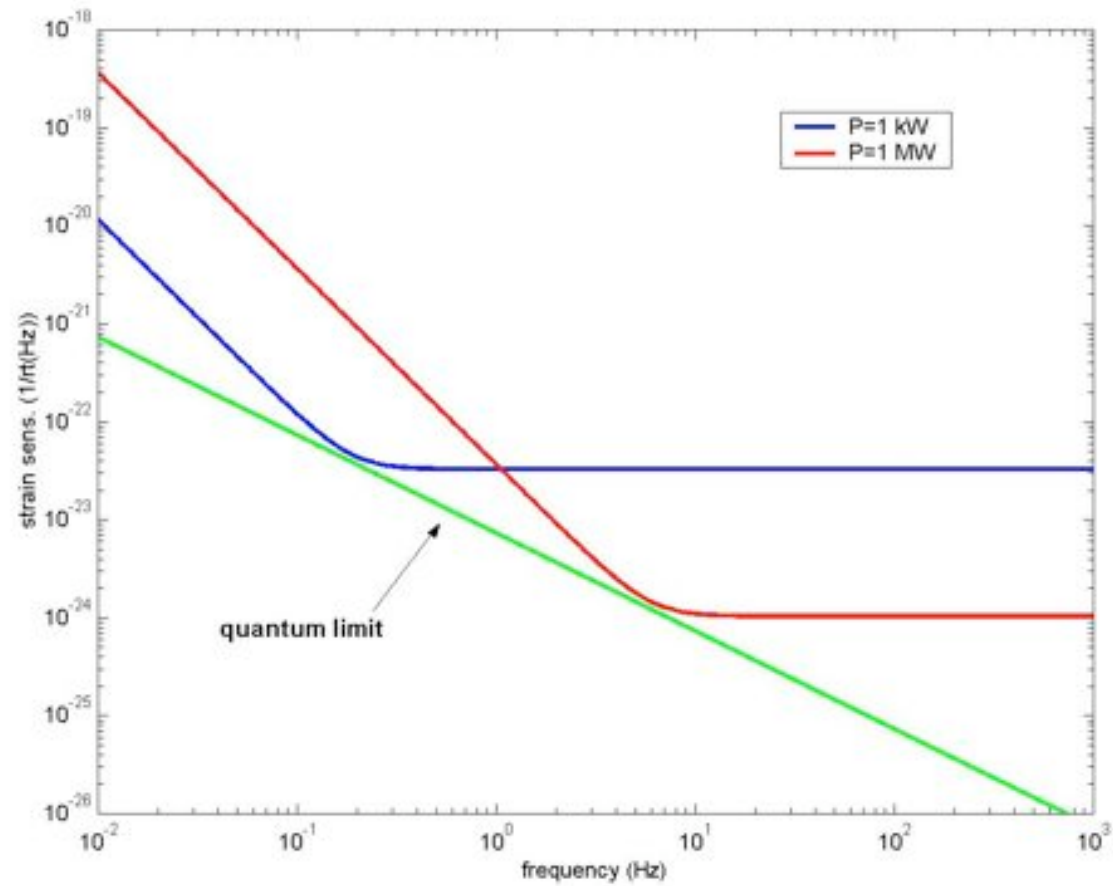
# Quantum Limit



# Quantum Limit

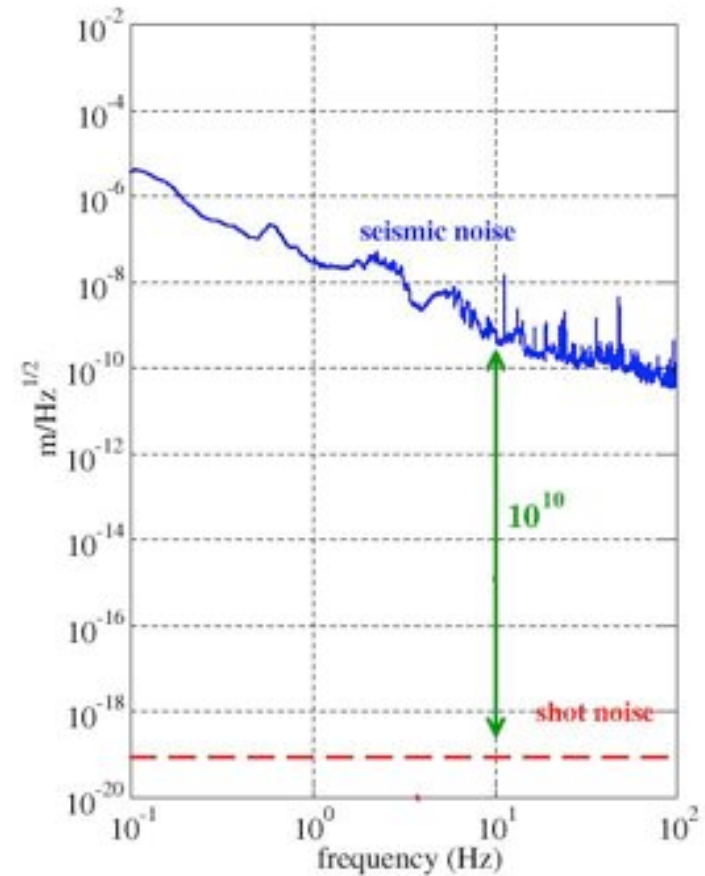
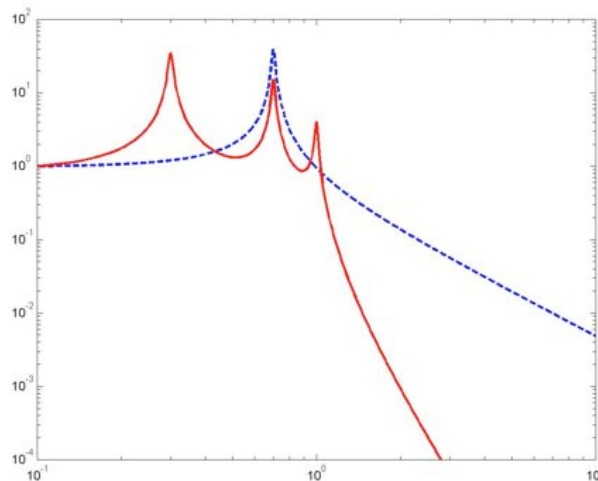
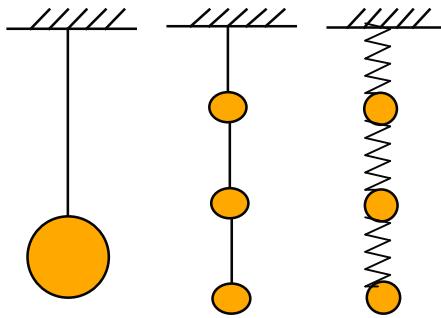


# Quantum Limit



## Rumore Sismico

- Specchi sospesi
- Uso di multipendoli
- Abbassare le frequenze caratteristiche
- Realizzare un isolamento su 6 d.o.f.

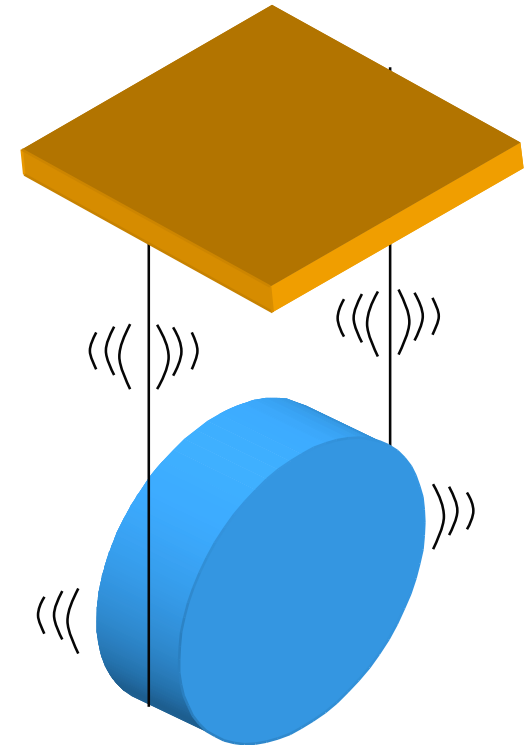
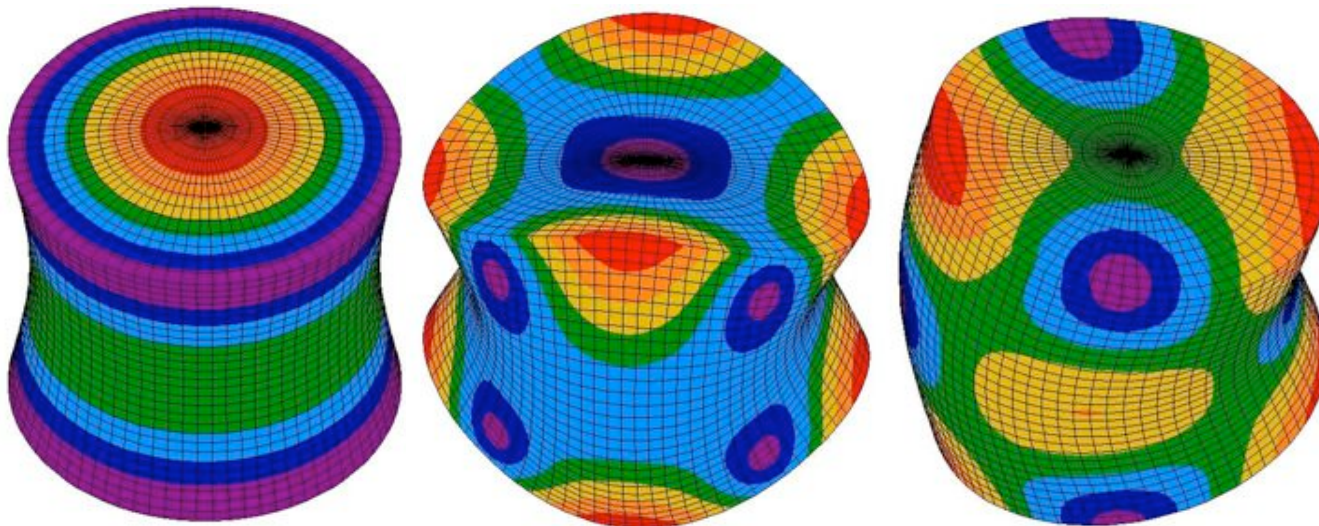


# Rumore Termico

## Teorema di fluttuazione e dissipazione:

$$\tilde{F}^2(f) = 4k_B T \beta(f)$$

- Rumore Termico: specchi, fili, penduli
- Possibile cura: ridurre le dissipazioni o raffreddare gli specchi sospesi



## Rumore indotto dalle fluttuazioni in frequenza del laser

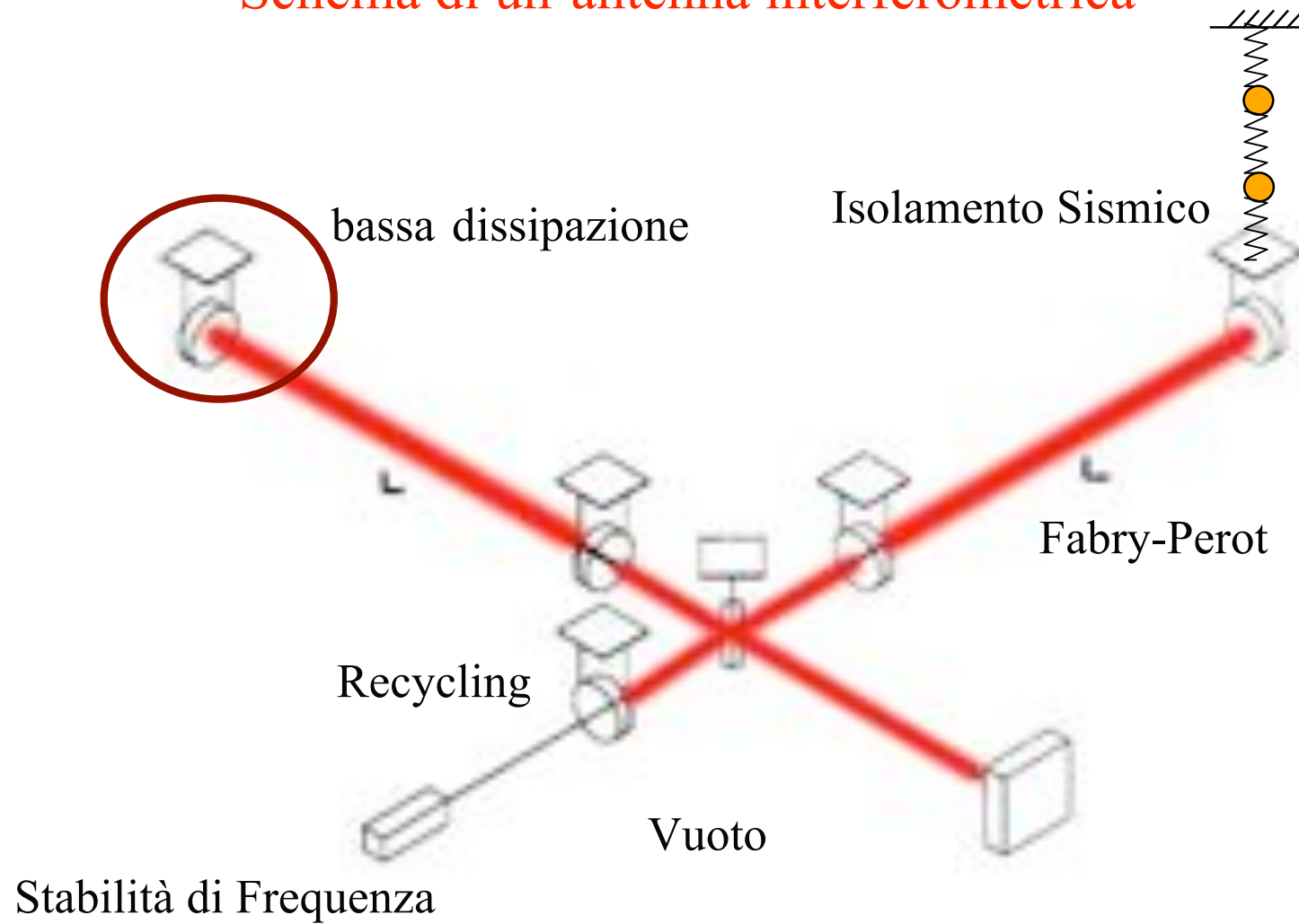
$$\Delta\phi = k\Delta L = \frac{\omega_{laser}\Delta L}{c} \longrightarrow \tilde{\phi} = k\Delta L = \frac{\tilde{\omega}_{laser}\Delta L}{c}$$

### PRECAUZIONI

- 1) Tecniche per rendere stabile la frequenza del laser
- 2) Si cerca di rendere il cammino ottico della luce il più uguale possibile (metri)

*Abbiamo tutti gli ingredienti per comprendere le ragioni  
del disegno di un'antenna interferometrica*

## Schema di un'antenna interferometrica

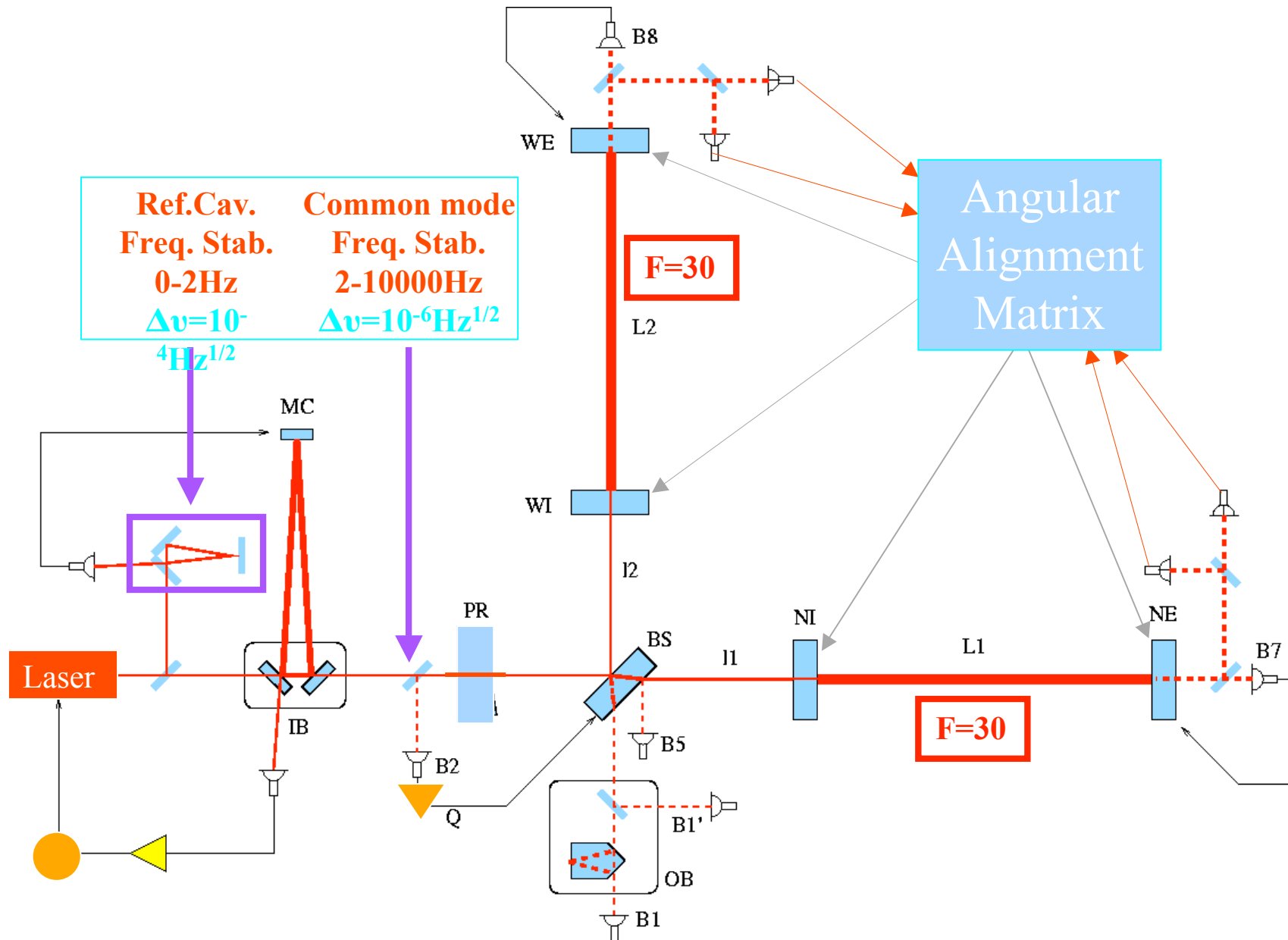


- **LAPP – Annecy**
- **INFN – Firenze-Urbino**
- **INFN – Frascati**
- **LMA – Lyon**
- **INFN – Napoli**
- **NIKHEF - Amsterdam**
- **OCA – Nice**
- **LAL – Orsay**
- **ESPCI – Paris**
- **INFN – Perugia**
- **INFN – Pisa**
- **INFN – Roma**

## **VIRGO at EGO Site**



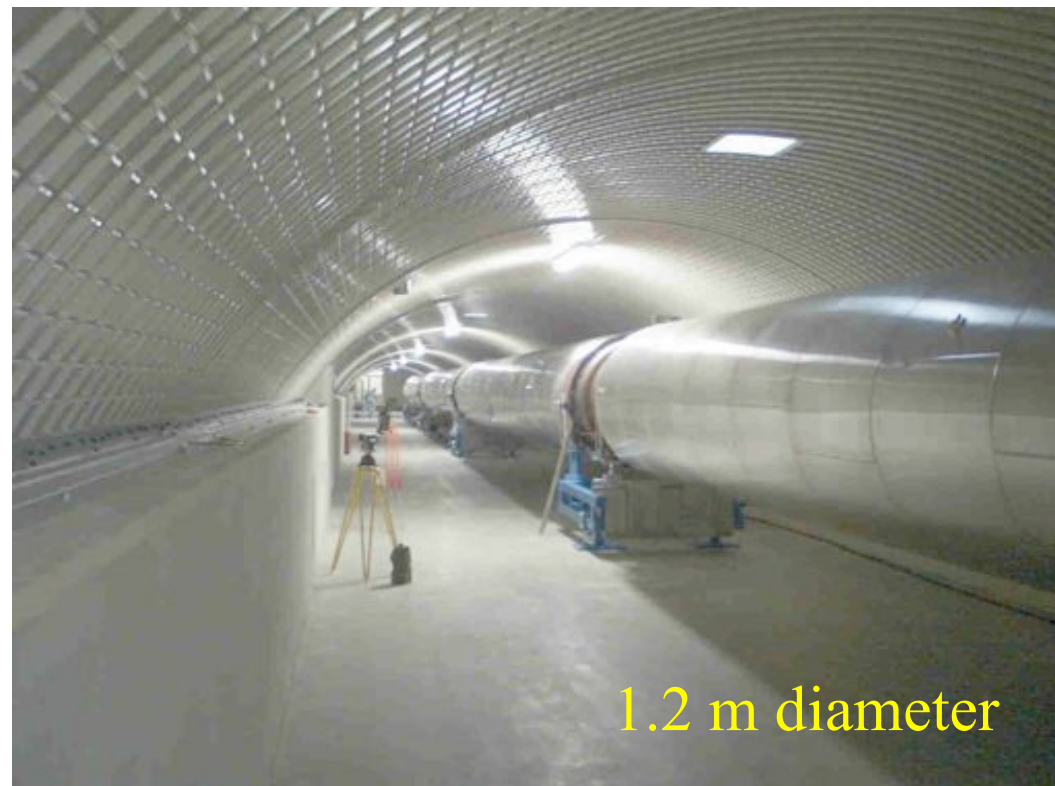
# Virgo Diagram





## Vacuum

- Requirements:
  - $10^{-9}$  mbar for  $H_2$
  - $10^{-14}$  mbar for hydrocarbons



## VIRGO Mirrors

35 cm diameter, 10 cm thickness

Absorption: 1 ppm

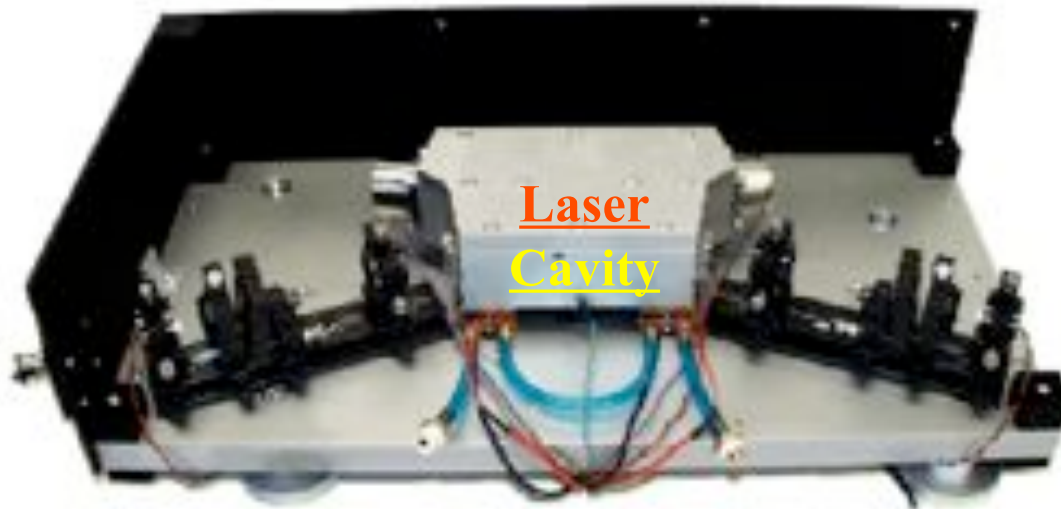
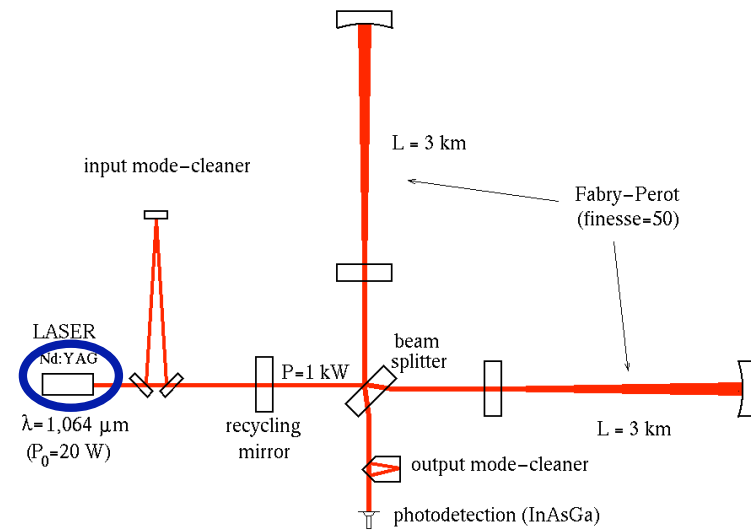
Reflection Losses: <5 ppm

Surface deformations: 0.01 micron





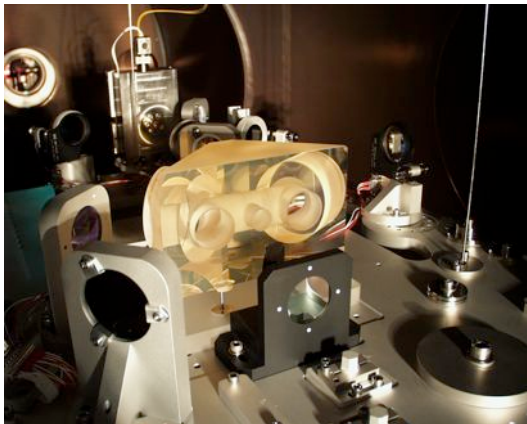
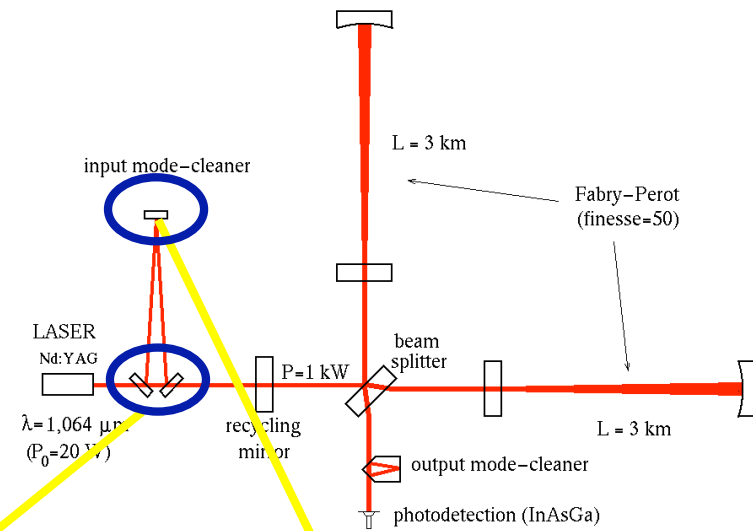
## Injection System



**Laser Source 20 W**  
**Nd:YVO4 laser ( $\lambda=1.064 \mu\text{m}$ )**

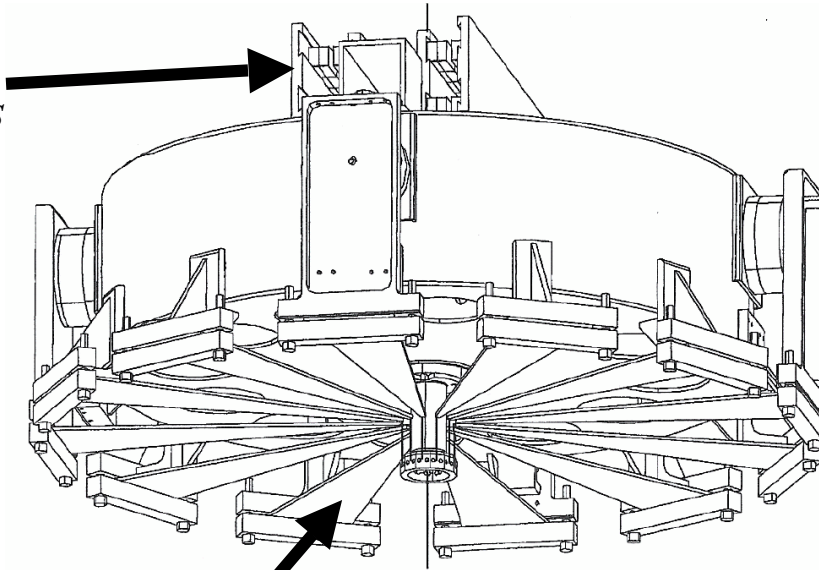


## Injection System



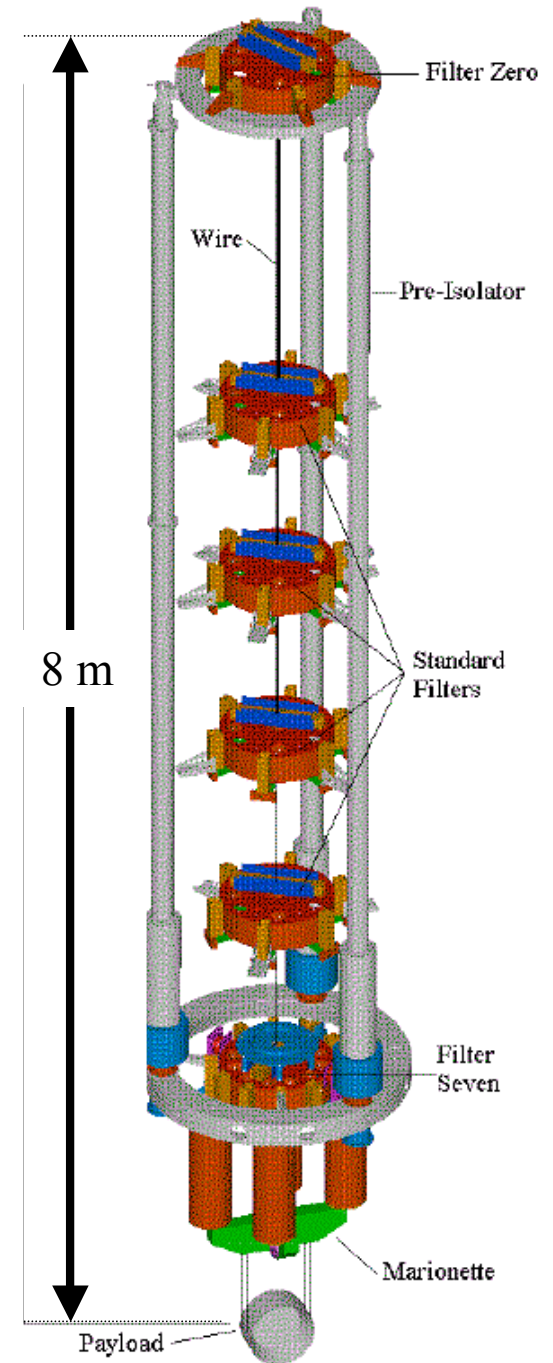
## Superattenuators

*Magnetic  
antisprings*



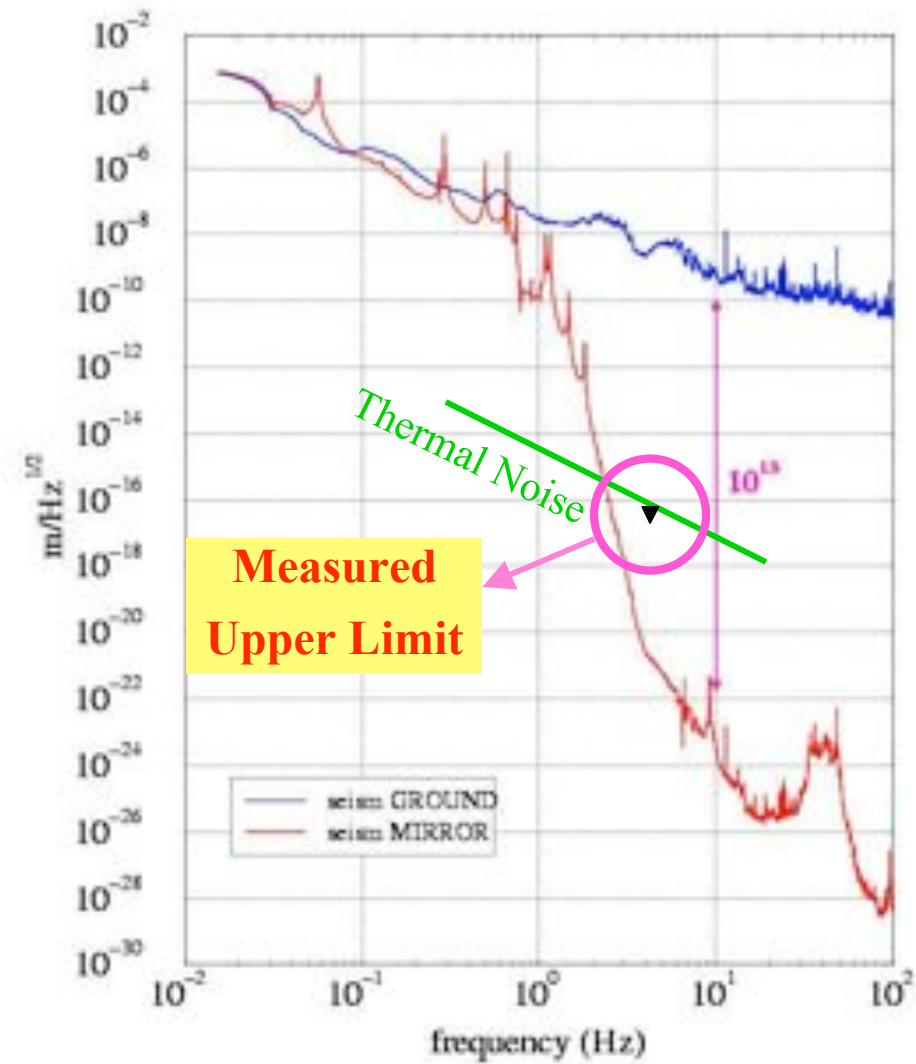
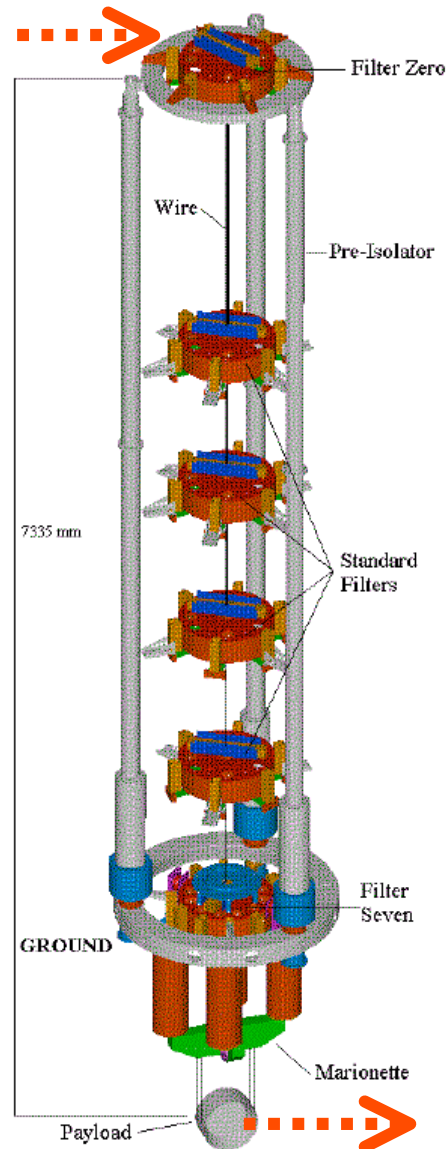
*Blade springs*

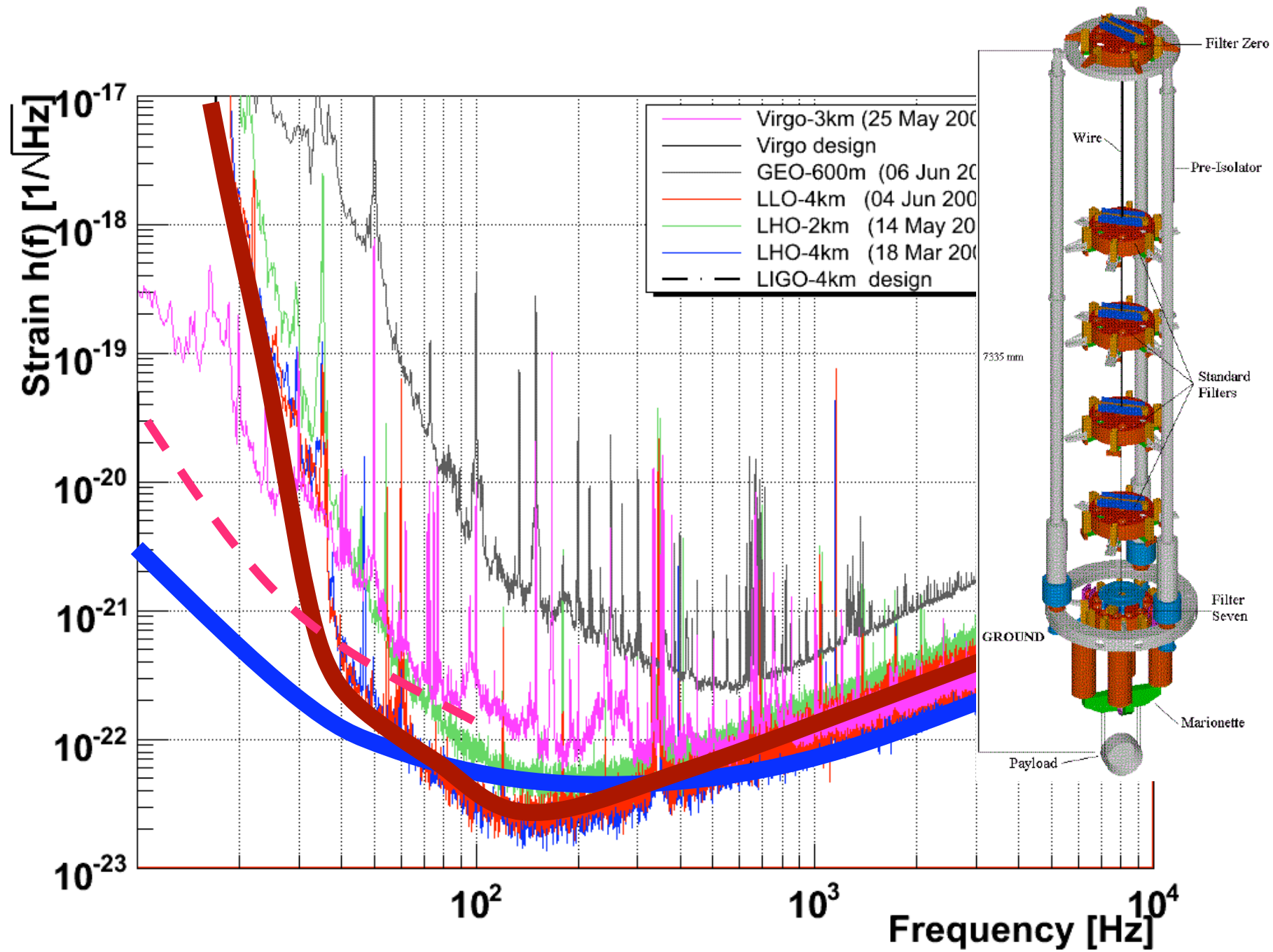
Extend the band down to  
a few Hz



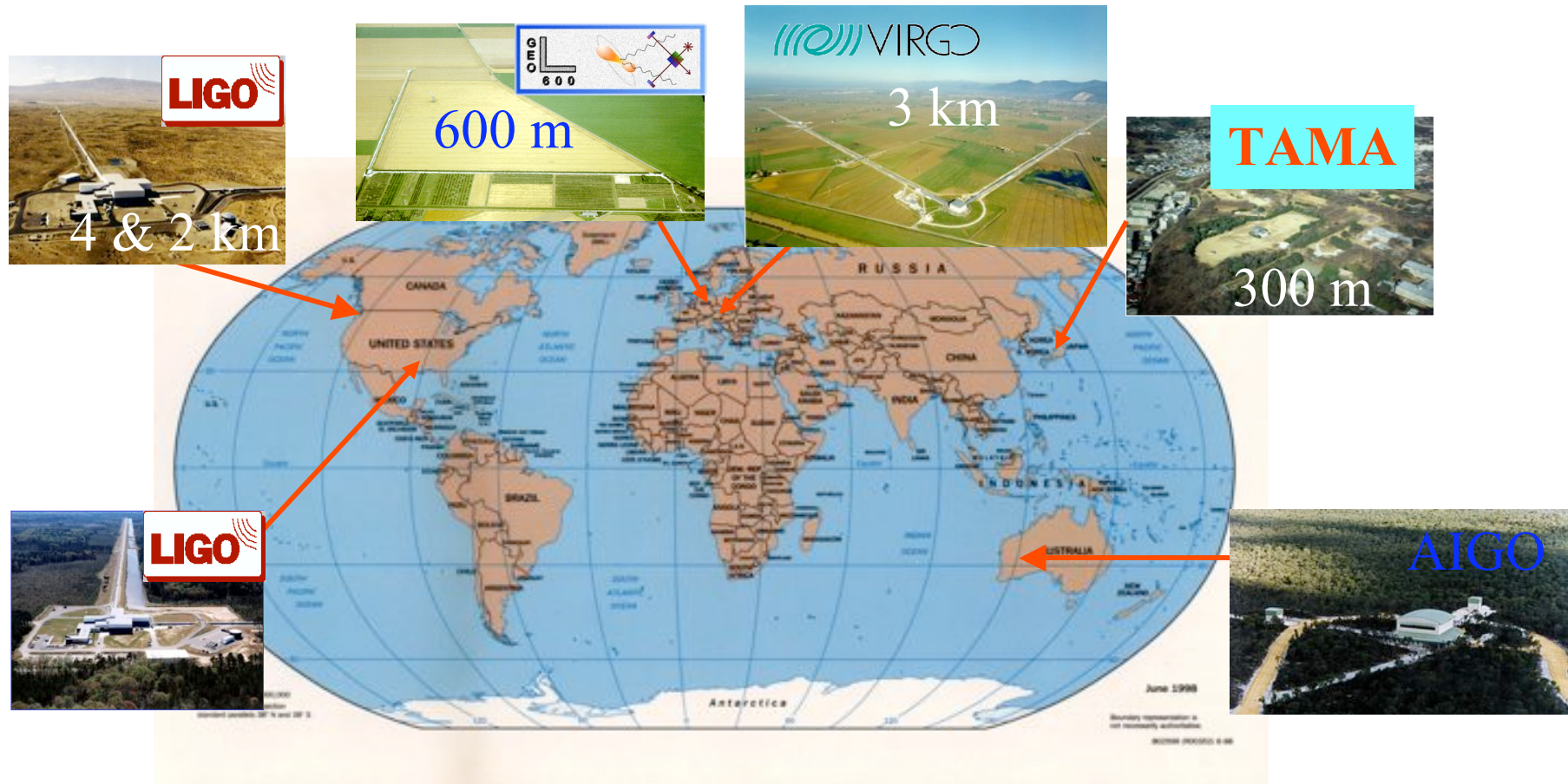


# Seismic Attenuation

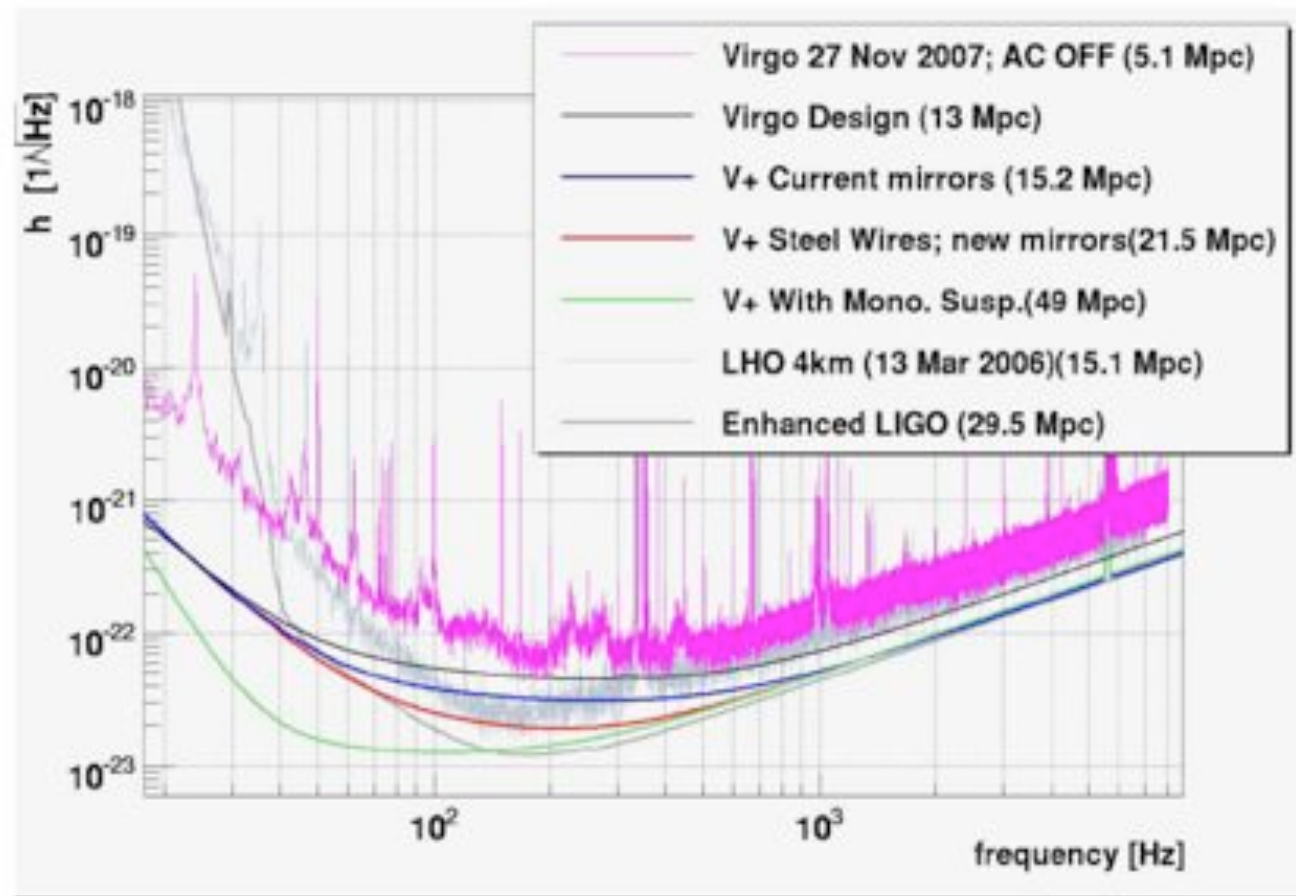




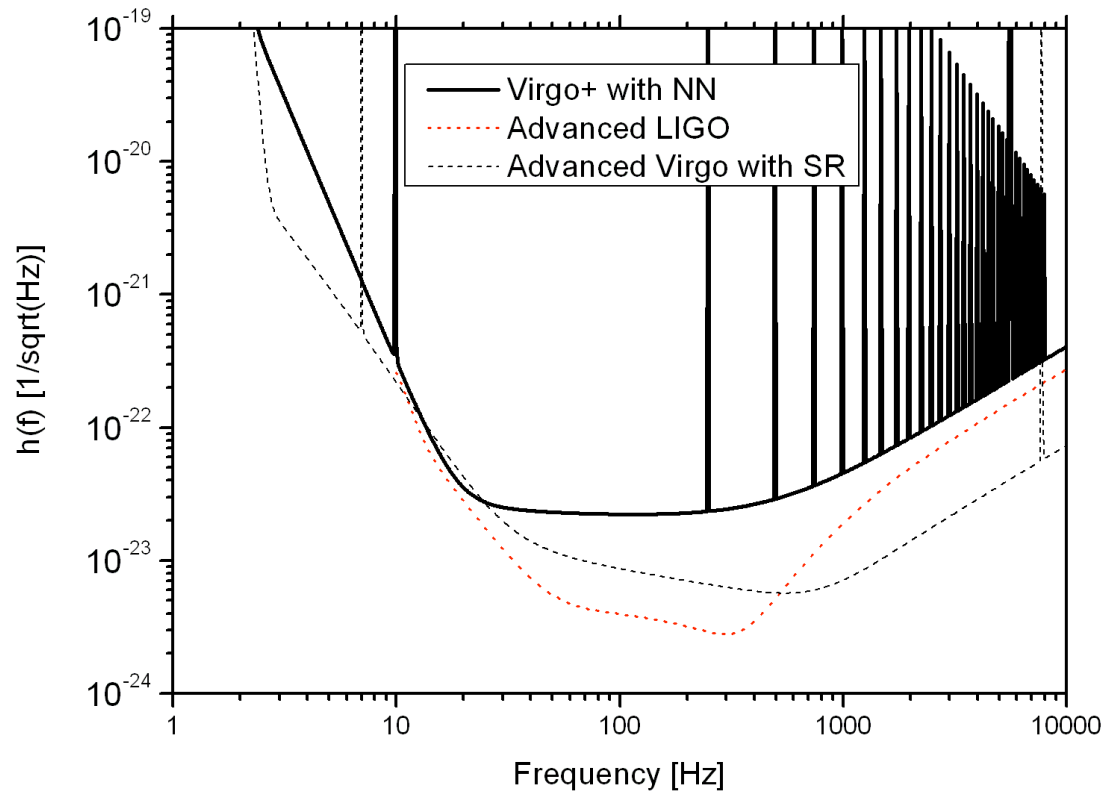
## Il Network di antenne interferometriche



## VIRGO+ POSSIBLE CURVES



# Next Generation



## Advanced LIGO (2013)

Higher power (10 W  $\rightarrow$  180 W)

New isolation system (active)

Fused silica suspension wires

40 kg fused silica mirrors

Signal recycling

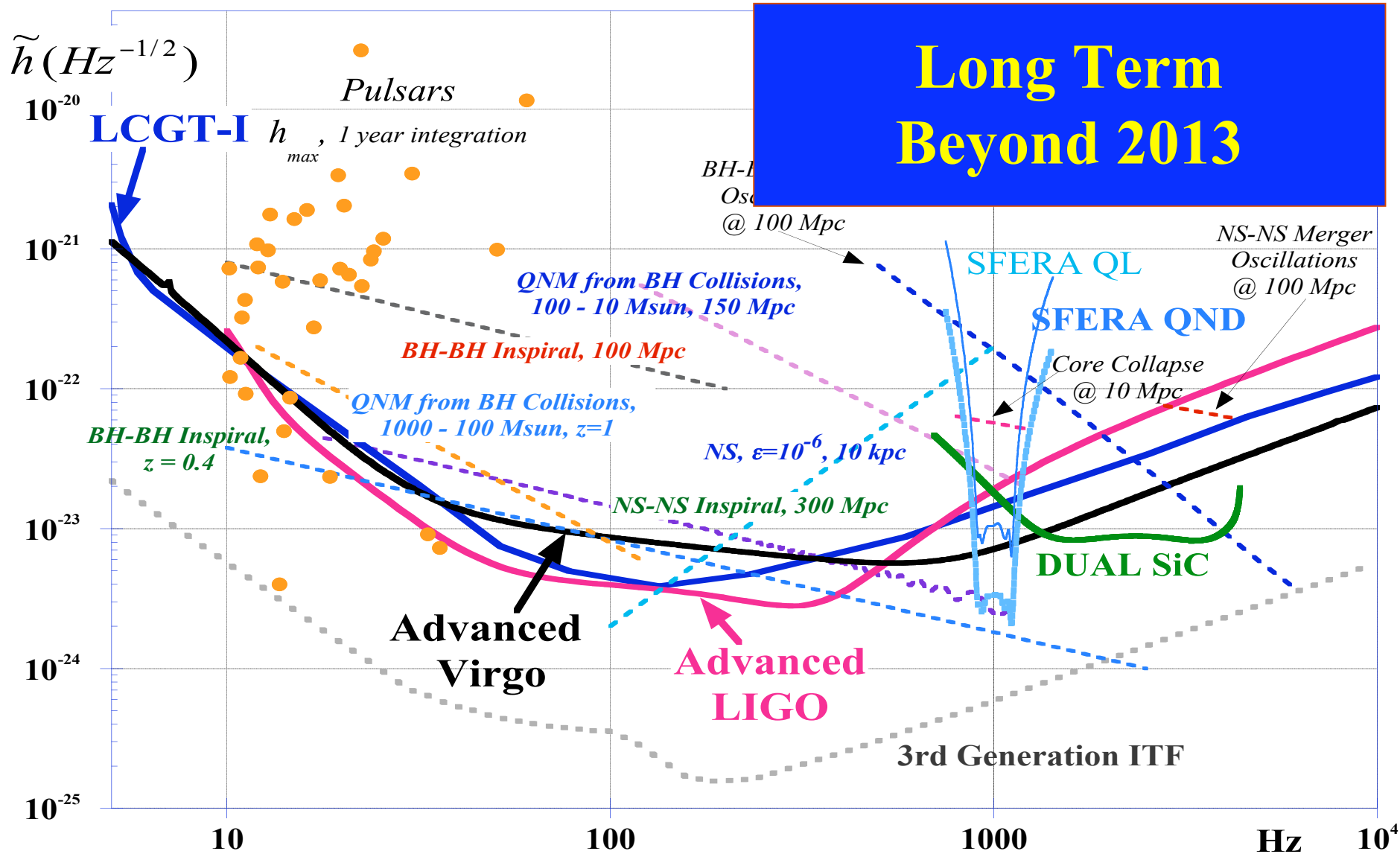
Similar program for Advanced VIRGO





Two underground 3 km itfs with mirrors at 20 K in the same vacuum system (Kamioka) – data taking 2012

LCGT project



A first detection is “sure”

**NS/NS  
detectable  
at 300 Mpc**

No detection means some serious trouble  
in fundamental physics

